Standard Test Method for
Dynamic Young's Modulus, Shear Modulus, and Poisson's Ratio for Advanced Ceramics by Impulse Excitation of Vibration[1]

This standard is issued under the fixed designation C 1259; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ϵ) indicates an editorial change since the last revision or reapproval.

1. Scope

1.1 This test method covers determination of the dynamic elastic properties of advanced ceramics at ambient temperatures. Specimens of these materials possess specific mechanical resonant frequencies that are determined by the elastic modulus, mass, and geometry of the test specimen. The dynamic elastic properties of a material can therefore be computed if the geometry, mass, and mechanical resonant frequencies of a suitable (rectangular, cylindrical, or disc geometry) test specimen of that material can be measured. Dynamic Young's modulus is determined using the resonant frequency in the flexural mode of vibration. The dynamic shear modulus, or modulus of rigidity, is found using torsional resonant vibrations. Dynamic Young's modulus and dynamic shear modulus are used to compute Poisson's ratio.

1.2 This test method measures the fundamental resonant frequency of test specimens of suitable geometry by exciting them mechanically by a singular elastic strike with an impulse tool. Specimen supports, impulse locations, and signal pick-up points are selected to induce and measure specific modes of the transient vibrations. A transducer (for example, contact accelerometer or non-contacting microphone) senses the resulting mechanical vibrations of the specimen and transforms them into electric signals. (See Fig. 1.) The transient signals are analyzed, and the fundamental resonant frequency is isolated and measured by the signal analyzer, which provides a numerical reading that is (or is proportional to) either the frequency or the period of the specimen vibration. The appropriate fundamental resonant frequencies, dimensions, and mass of the specimen are used to calculate dynamic Young's modulus, dynamic shear modulus, and Poisson's ratio.

1.3 Although not specifically described herein, this test method can also be performed at cryogenic and high temperatures with suitable equipment modifications and appropriate
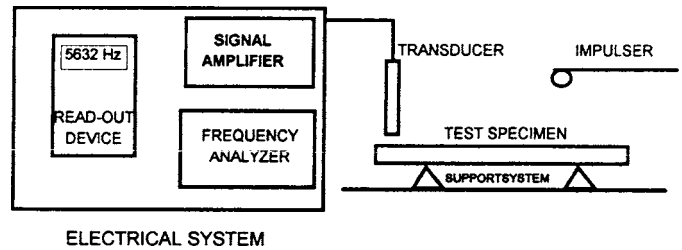

FIG. 1 Block Diagram of Typical Test Apparatus

modifications to the calculations to compensate for thermal expansion, in accordance with sections 9.2, 9.3, and 10.4 of C 1198.

1.4 Where possible, the procedures, sample specifications, and calculations in this test method are consistent with Test Methods C 623, C 747, C 848, and C 1198.

1.5 This test method uses test specimens in bar, rod, and disc geometries. The rod and bar geometries are described in the main body. The disc geometry is addressed in Annex A1.

1.6 A modification of this test method can be used for quality control and nondestructive evaluation, using changes in resonant frequency to detect variations in specimen geometry and mass and internal flaws in the specimen. (See 5.5.)

1.7 The values stated in SI units are to be regarded as the standard.

1.8 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory limitations prior to use.*

2. Referenced Documents

2.1 *ASTM Standards:* [2]

C 372 Test Method for Linear Thermal Expansion of Porcelain Enamel and Glaze Frits and Fired Ceramic Whiteware Products by the Dilatometer Method

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[1] This test method is under the jurisdiction of ASTM Committee C28 on Advanced Ceramics and is the direct responsibility of Subcommittee C28.01 on Mechanical Properties and Performance

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[2] For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

3. Terminology

3.1 *Definitions*—The definitions of terms relating to mechanical testing appearing in Terminology E 6 should be considered as applying to the terms used in this test method. The definitions of terms relating to advanced ceramics appearing in Terminology C 1145 should be considered as applying to the terms used in this test method. Directly pertinent definitions as listed in Terminologies E 6, C 1145, and D 4092 are shown in the following paragraphs with the appropriate source given in brackets.

3.1.1 *advanced ceramic*, *n*—a highly engineered, high-performance, predominately nonmetallic, inorganic, ceramic material having specific functional attributes. (**C 1145**)

3.1.2 *dynamic mechanical measurement*, *n*—a technique in which either the modulus or damping, or both, of a substance under oscillatory load or displacement is measured as a function of temperature, frequency, or time, or combination thereof. (**D 4092**)

3.1.3 *elastic limit* [FL^{-2}], *n*—the greatest stress that a material is capable of sustaining without permanent strain remaining upon complete release of the stress. (**E 6**)

3.1.4 *elastic modulus* [FL^{-2}], *n*—the ratio of stress to strain below the proportional limit. (**E 6**)

3.1.5 *Poisson's ratio* (µ) [nd], *n*—the absolute value of the ratio of transverse strain to the corresponding axial strain resulting from uniformly distributed axial stress below the proportional limit of the material.

3.1.5.1 *Discussion*—In isotropic materials, Young's Modulus (*E*), shear modulus (*G*), and Poisson's ratio (µ) are related by the following equation:

$$\mu = (E/2G) - 1 \qquad (1)$$

3.1.6 *proportional limit* [FL^{-2}], *n*—the greatest stress that a material is capable of sustaining without deviation from proportionality of stress to strain (Hooke's law). (**E 6**)

3.1.7 *shear modulus (G)* [FL^{-2}], *n*—the elastic modulus in shear or torsion. Also called modulus of rigidity or torsional modulus. (**E 6**)

3.1.8 *Young's modulus (E)* [FL^{-2}], *n*—the elastic modulus in tension or compression. (**E 6**)

3.2 *Definitions of Terms Specific to This Standard:*

3.2.1 *antinodes*, *n*—two or more locations that have local maximum displacements, called antinodes, in an unconstrained slender rod or bar in resonance. For the fundamental flexure resonance, the antinodes are located at the two ends and the center of the specimen.

3.2.2 *elastic*, *adj*—the property of a material such that an application of stress within the elastic limit of that material making up the body being stressed will cause an instantaneous and uniform deformation, which will be eliminated upon removal of the stress, with the body returning instantly to its original size and shape without energy loss. Most advanced ceramics conform to this definition well enough to make this resonance test valid.

3.2.3 *flexural vibrations*, *n*—the vibrations that occur when the displacements in a slender rod or bar are in a plane normal to the length dimension.

3.2.4 *homogeneous*, *adj*—the condition of a specimen such that the composition and density are uniform, so that any smaller specimen taken from the original is representative of the whole. Practically, as long as the geometrical dimensions of the test specimen are large with respect to the size of individual grains, crystals, components, pores, or microcracks, the body can be considered homogeneous.

3.2.5 *in-plane flexure*, *n*—for rectangular parallelepiped geometries, a flexure mode in which the direction of displacement is in the major plane of the test specimen.

3.2.6 *isotropic*, *adj*—the condition of a specimen such that the values of the elastic properties are the same in all directions in the material. Advanced ceramics are considered isotropic on a macroscopic scale, if they are homogeneous and there is a random distribution and orientation of phases, crystallites, components, pores, or microcracks.

3.2.7 *nodes*, *n*—one or more locations in a slender rod or bar in resonance having a constant zero displacement. For the fundamental flexural resonance of such a rod or bar, the nodes are located at 0.224 *L* from each end, where *L* is the length of the specimen.

3.2.8 *out-of-plane flexure*, *n*—for rectangular parallelepiped geometries, a flexure mode in which the direction of displacement is perpendicular to the major plane of the test specimen.

3.2.9 *resonant frequency*, *n*—naturally occurring frequencies of a body driven into flexural, torsional, or longitudinal vibration that are determined by the elastic modulus, mass, and dimensions of the body. The lowest resonant frequency in a given vibrational mode is the fundamental resonant frequency of that mode.

3.2.10 *slender rod or bar*, *n*—in dynamic elastic property testing, a specimen whose ratio of length to minimum cross-sectional dimension is at least 5 and preferably in the range of 20 to 25.

3.2.11 *torsional vibrations*, *n*—the vibrations that occur when the oscillations in each cross-sectional plane of a slender rod or bar are such that the plane twists around the length dimension axis.

4. Summary of Test Method

4.1 This test method measures the fundamental resonant frequency of test specimens of suitable geometry (bar, rod, or disc) by exciting them mechanically by a singular elastic strike with an impulse tool. A transducer (for example, contact accelerometer or non-contacting microphone) senses the resulting mechanical vibrations of the specimen and transforms them into electric signals. Specimen supports, impulse locations, and signal pick-up points are selected to induce and measure specific modes of the transient vibrations. The signals are analyzed, and the fundamental resonant frequency is isolated and measured by the signal analyzer, which provides a numerical reading that is (or is proportional to) either the frequency or the period of the specimen vibration. The appropriate fundamental resonant frequencies, dimensions, and mass of the specimen are used to calculate dynamic Young's modulus, dynamic shear modulus, and Poisson's ratio.

5. Significance and Use

5.1 This test method may be used for material development, characterization, design data generation, and quality control purposes.

5.2 This test method is specifically appropriate for determining the modulus of advanced ceramics that are elastic, homogeneous, and isotropic (**1**).[3]

5.3 This test method addresses the room temperature determination of dynamic moduli of elasticity of slender bars (rectangular cross-section) and rods (cylindrical). Flat plates and disks may also be measured similarly, but the required equations for determining the moduli are not addressed herein.

5.4 This dynamic test method has several advantages and differences from static loading techniques and from resonant techniques requiring continuous excitation.

5.4.1 The test method is nondestructive in nature and can be used for specimens prepared for other tests. The specimens are subjected to minute strains; hence, the moduli are measured at or near the origin of the stress-strain curve, with the minimum possibility of fracture.

5.4.2 The impulse excitation test uses an impact tool and simple supports for the test specimen. There is no requirement for complex support systems that require elaborate setup or alignment.

5.5 This technique can be used to measure resonant frequencies alone for the purposes of quality control and acceptance of test specimens of both regular and complex shapes. A range of acceptable resonant frequencies is determined for a specimen with a particular geometry and mass. Deviations in specimen dimensions or mass and internal flaws (cracks, delaminations, inhomogeneities, porosity, etc) will change the resonant frequency for that specimen. Any specimen with a resonant frequency falling outside the prescribed frequency range is rejected. The actual modulus of each specimen need not be determined as long as the limits of the selected frequency range are known to include the resonant frequency that the specimen must possess if its geometry and mass and internal structure are within specified tolerances. The technique is particularly suitable for testing specimens with complex geometries (other than parallelepipeds, cylinders/rods, or discs) that would not be suitable for testing by other procedures. This is similar to the evaluation method described in Guide E 2001.

5.6 If a thermal treatment or an environmental exposure affects the elastic response of the test specimen, this test method may be suitable for the determination of specific effects of thermal history, environment exposure, etc. Specimen descriptions should include any specific thermal treatments or environmental exposures that the specimens have received.

6. Interferences

6.1 The relationships between resonant frequency and dynamic modulus presented herein are specifically applicable to homogeneous, elastic, isotropic materials.

6.1.1 This method of determining the moduli is applicable to composite ceramics and inhomogeneous materials only with careful consideration of the effect of inhomogeneities and anisotropy. The character (volume fraction, size, morphology, distribution, orientation, elastic properties, and interfacial bonding) of the reinforcement and inhomogeneities in the specimens will have a direct effect on the elastic properties of the specimen as a whole. These effects must be considered in interpreting the test results for composites and inhomogeneous materials.

6.1.2 The procedure involves measuring transient elastic vibrations. Materials with very high damping capacity may be difficult to measure with this technique if the vibration damps out before the frequency counter can measure the signal (commonly within three to five cycles).

6.1.3 If specific surface treatments (coatings, machining, grinding, etching, etc.) change the elastic properties of the near-surface material, there will be accentuated effects on the properties measured by this flexural method, as compared to static/bulk measurements by tensile or compression testing.

6.1.4 The test method is not satisfactory for specimens that have major discontinuities, such as large cracks (internal or surface) or voids.

6.2 This test method for determining moduli is limited to specimens with regular geometries (rectangular parallelepiped, cylinders, and discs) for which analytical equations are available to relate geometry, mass, and modulus to the resonant vibration frequencies. The test method is not appropriate for determining the elastic properties of materials that cannot be fabricated into such geometries.

6.2.1 The analytical equations assume parallel and concentric dimensions for the regular geometries of the specimen. Deviations from the specified tolerances for the dimensions of

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[3] The boldface numbers in parentheses refer to the list of references at the end of this test method.

the specimens will change the resonant frequencies and introduce error into the calculations.

6.2.2 Edge treatments such as chamfers or radii are not considered in the analytical equations. Edge chamfers on flexure bars prepared according to Test Method C 1161 will change the resonant frequency of the test bars and introduce error into the calculations of the dynamic modulus. It is recommended that specimens for this test method not have chamfered or rounded edges. Alternately, if narrow rectangular specimens with chamfers or edge radii are tested, then the procedures in Annex A2 should be used to correct the calculated Young's modulus, E.

6.2.3 For specimens with as-fabricated and rough or uneven surfaces, variations in dimensions can have a significant effect in the calculations. For example, in the calculation of dynamic modulus, the modulus value is inversely proportional to the cube of the thickness. Uniform specimen dimensions and precise measurements are essential for accurate results.

6.3 The test method assumes that the specimen is vibrating freely, with no significant restraint or impediment. Specimen supports should be designed and located properly in accordance with 9.3.1, 9.4.1, and 9.5.1 so the specimen can vibrate freely in the desired mode. In using direct contact transducers, the transducer should be positioned away from antinodes and with minimal force to avoid interference with free vibration. With noncontacting transducers, the maximum sensitivity is accomplished by placing the transducer at an antinode.

6.4 Proper location of the impulse point and transducer is important in introducing and measuring the desired vibration mode. The locations of the impulse point and transducer should not be changed in multiple readings; changes in position may develop and detect alternate vibration modes. In the same manner, the force used in impacting should be consistent in multiple readings.

6.5 If the frequency readings are not repeatable for a specific set of impulse and transducer locations on a specimen, it may be because several different modes of vibration are being developed and detected in the test. The geometry of the test bar and desired vibration mode should be evaluated and used to identify the nodes and antinodes of the desired vibrations. More consistent measurements may be obtained if the impulse point and transducer locations are shifted to induce and measure the single desired mode of vibration.

7. Apparatus

7.1 Apparatus suitable for accurately detecting, analyzing, and measuring the fundamental resonant frequency or period of a vibrating free-free beam is used. The test apparatus is shown in Fig. 1. It consists of an impulser, a suitable pickup transducer to convert the mechanical vibration into an electrical signal, an electronic system (consisting of a signal conditioner/amplifier, a signal analyzer, and a frequency readout device), and a support system. Commercial instrumentation is available that measures the frequency or period of the vibrating specimen. See Note 1.

NOTE 1—One such instrument is the Grindosonic instrument, manufactured by J. W. Lemmens, Inc., 10801 Pear Tree Lane, St. Louis, MO 63074.
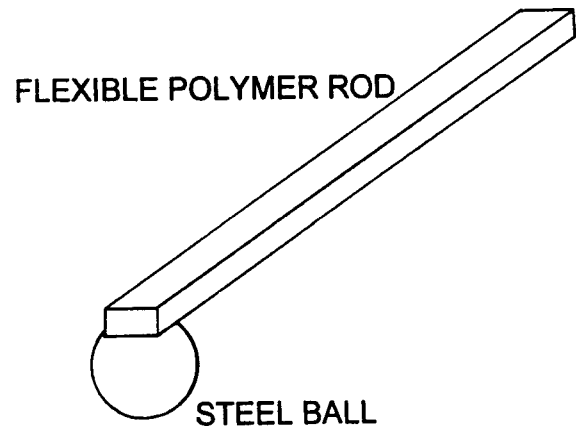

FIG. 2 Diagram of Typical Impulser for Small Specimens

7.2 *Impulser*—The exciting impulse is imparted by lightly striking the specimen with a suitable implement. This implement should have most of its mass concentrated at the point of impact and have mass sufficient to induce a measurable mechanical vibration, but not so large as to displace or damage the specimen physically. In practice, the size and geometry of the impulser depends on the size and weight of the specimen and the force needed to produce vibration. For commonly tested geometries (small bars, rods, and discs) in advanced ceramics, an example of such an impulser would be a steel sphere 0.5 cm in diameter glued to the end of a flexible 10–cm long polymer rod. (See Fig. 2.) An alternate impulser would be a solid metal, ceramic, or polymer sphere (0.1 to 1.0 cm in diameter) dropped on the specimen through a guide tube to ensure proper impulse position.

7.3 *Signal Pickup*—Signal detection can be via transducers in direct contact with the specimen or by non-contact transducers. Contact transducers are commonly accelerometers using piezoelectric or strain gage methods to measure the vibration. Non-contact transducers are commonly acoustic microphones, but they may also use laser, magnetic, or capacitance methods to measure the vibration. The frequency range of the transducer shall be sufficient to measure the expected frequencies of the specimens of interest. A suitable range would be 100 Hz to 50 kHz for most advanced ceramic test specimens. (Smaller and stiffer specimens vibrate at higher frequencies.) The frequency response of the transducer across the frequency range of interest shall have a bandwidth of at least 10% of the maximum measured frequency before -3 dB power loss occurs.

7.4 *Electronic System*—The electronic system consists of a signal conditioner/amplifier, signal analyzer, and a frequency readout device. The system should have accuracy and precision sufficient to measure the frequencies of interest to an accuracy of 0.1%. The signal conditioner/amplifier should be suitable to power the transducer and provide and appropriate amplified signal to the signal analyzer. The signal analysis system consists of a frequency counting device and a readout device. Appropriate devices are frequency counter systems with storage capability or digital storage oscilloscopes with a frequency counter module. With the digital storage oscilloscope, a Fast Fourier Transform signal analysis system may be useful for

analyzing more complex waveforms and identifying the fundamental resonant frequency.

7.5 *Support System*—The support shall serve to isolate the specimen from extraneous vibration without restricting the desired mode of specimen vibration. Appropriate materials should be stable at the test temperatures. Support materials can be either soft or rigid for ambient conditions. Examples of soft materials would be a compliant elastomeric material, such as polyurethane foam strips. Such foam strips would have simple flat surfaces for the specimen to rest on. Rigid materials, such as metal or ceramic, should have sharp knife edges or cylindrical surfaces on which the specimen should rest. The rigid supports should be resting on isolation pads to prevent ambient vibrations from being picked up by the transducer. Wire suspension can also be used. Specimens shall be supported along node lines appropriate for the desired vibration in the locations described in Section 8.

8. Test Specimen

8.1 The specimens shall be prepared so that they are either rectangular or circular in cross section. Either geometry can be used to measure both dynamic Young's modulus and dynamic shear modulus. Although the equations for computing shear modulus with a cylindrical specimen are both simpler and more accurate than those used with a rectangular bar, experimental difficulties in obtaining torsional resonant frequencies for a cylindrical specimen usually preclude its use for determining shear modulus. (See Annex A1 for disc specimens.)

8.2 Resonant frequencies for a given specimen are functions of the specimen dimensions as well as its mass and moduli; dimensions should therefore be selected with this relationship in mind. The selection of size shall be made so that, for an estimated modulus, the resonant frequencies measured will fall within the range of frequency response of the transducers and electronics used. For a slender rod, the ratio of length to minimum cross-sectional dimension shall have a value of at least 10. However, a ratio of approximately $20 \approx 25$ is preferred for ease in calculation. For shear modulus measurements of rectangular bars, a ratio of width to thickness of 5 or greater is recommended for minimizing experimental difficulties.

8.3 All surfaces on the rectangular specimen shall be flat. Opposite surfaces across the length and width shall be parallel within 0.01 mm or ±0.1%, whichever is greater. Opposite surfaces across the thickness shall be parallel within 0.002 mm or ±0.1%, whichever is greater. The cylindrical specimen shall be round and constant in diameter within 0.002 mm or ±0.1%, whichever is greater.

8.4 Test specimen mass shall be determined within ± 0.1% or 10 mg, whichever is greater.

8.5 Test specimen length shall be measured to within ± 0.1%. Test specimen cross sectional dimensions (thickness and width in rectangular beams; diameter in cylindrical rods) shall be measured within ± 0.1% or 0.01 mm at three equally-spaced locations along the length and an average value determined.

8.6 Porous materials and those susceptible to hydration should be dried in air at 120°C in a drying oven until the mass is constant (less than 0.1% or 10 mg difference in measured mass with 30 min of additional drying).

8.7 It is recommended that the laboratory obtain and maintain an internal reference specimen with known and recorded fundamental resonant frequencies in flexure and torsion. The reference specimen should be used to check and confirm the operation of the test system on a regular basis. It can also be used to train operators in the proper test setup and test procedure. The reference specimen can be a standard ceramic material (alumina, silicon carbide, zirconia, etc.) or it may be of a similar size, composition, and microstructure to the types of ceramic specimens commonly tested at the laboratory. The reference specimen must meet the size, dimensional tolerances, and surface finish requirements of Section 8.

9. Procedure

9.1 Activate all electrical equipment, and allow it to stabilize according to the manufacturer's recommendations.

9.2 Use a test specimen established as a verification/calibration standard to verify the equipment response and accuracy.

9.3 *Fundamental Flexural Resonant Frequency (Out-of-Plane Flexure)*:

9.3.1 Place the specimen on the supports located at the fundamental nodal points (0.224 L from each end; see Fig. 3).

9.3.2 Determine the direction of maximum sensitivity for the transducer. Orient the transducer so that it will detect the desired vibration.

9.3.2.1 *Direct Contact Transducers*—Place the transducer in contact with the test specimen to pick up the desired vibration. If the transducer is placed at an antinode (location of maximum displacement), it may mass load the specimen and modify the natural vibration. The transducer should preferably be placed only as far from the nodal points as necessary to obtain a reading (see Fig. 3). This location will minimize the damping effect from the contacting transducer. The transducer contact force should be consistent, with good response and minimal interference with the free vibration of the specimen.

9.3.2.2 *Non-Contact Transducers*—Place the non-contact transducer over an antinode point and close enough to the test specimen to pick up the desired vibration, but not so close as to interfere with the free vibration (see Fig. 3).

9.3.3 Strike the specimen lightly and elastically, either at the center of the specimen or at the opposite end of the specimen from the detecting transducer (see Fig. 3).

9.3.4 Record the resultant reading, and repeat the test until a recommended ten readings are obtained that lie within ±10 % of the mean. The round-robin interlaboratory study (12.2) showed that data points significantly (>10 %) out of range were measurements of spurious vibration modes or secondary harmonics. If ten readings cannot be taken, a minimum of five readings that lie within ±10 % of the mean shall be required for estimating the mean. Use the mean of these readings to determine the fundamental resonant frequency in flexure.

9.4 *Fundamental Flexural Resonant Frequency (In-Plane Flexure)*:

9.4.1 This procedure is the same as that above (9.3), except that the direction of vibration is in the major plane of the specimen. This measurement can be performed in two ways. In
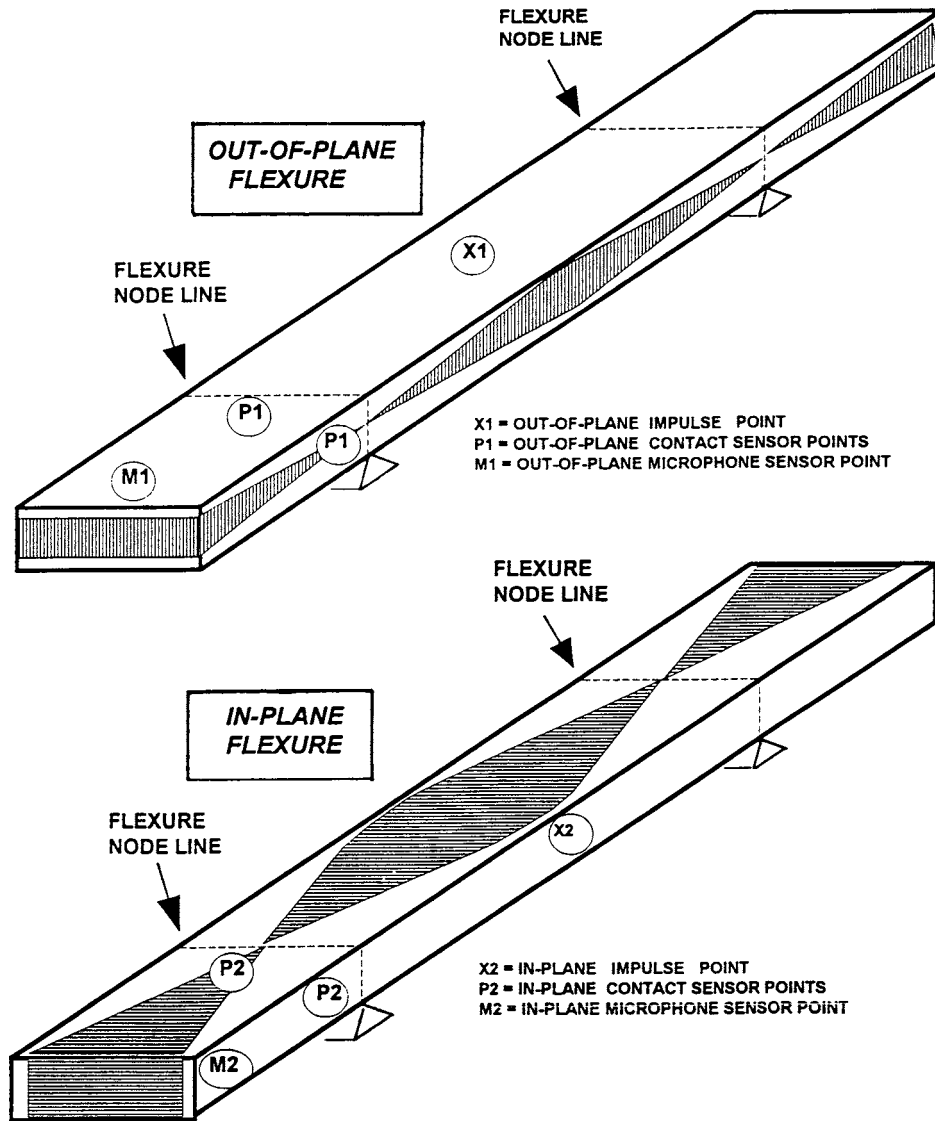
FIG. 3 Rectangular Specimens Tested for In-Plane and Out-of-Plane Flexure

one case, move the transducer and impulser 90° around the long axis of the test specimen to introduce and detect vibrations in the major plane (see Fig. 3). In the alternate method, rotate the test bar 90° around its long axis and reposition it on the specimen supports. Transpose the width and thickness dimensions in the calculations. For homogeneous, isotropic materials, the calculated moduli should be the same as the moduli calculated from the out-of-plane frequency. The comparison of in-plane and out-of-plane frequency measurements can thus be used as a cross check of experimental methods and calculations.

9.5 *Fundamental Torsional Resonant Frequency*:

9.5.1 Support the specimen at the midpoint of its length and width (the torsional nodal planes) (see Fig. 4).

9.5.2 Locate the transducer at one quadrant of the specimen, preferably at approximately 0.224 L from one end and toward the edge. This location is a nodal point of flexural vibration and will minimize the possibility of detecting a spurious flexural mode (see Fig. 4).

9.5.3 Strike the specimen on the quadrant diagonally opposite the transducer, again at 0.224 L from the end and near the edge. Striking at a flexural nodal point will minimize the possibility of exciting a flexural mode of vibration (see Fig. 4).

9.5.4 Record the resultant reading, and repeat the test until a recommended ten readings are obtained that lie within ±10 % of the mean. The round-robin interlaboratory study (12.2) showed that data points significantly (>10%) out of range were measurements of spurious vibration modes or secondary harmonics. If ten readings cannot be taken, a minimum of five readings that lie within ±10 % of the mean shall be required for estimating the mean. Use the mean of these readings to determine the fundamental resonant frequency in torsion.

10. Calculation

10.1 *Dynamic Young's Modulus (2, 3)*:

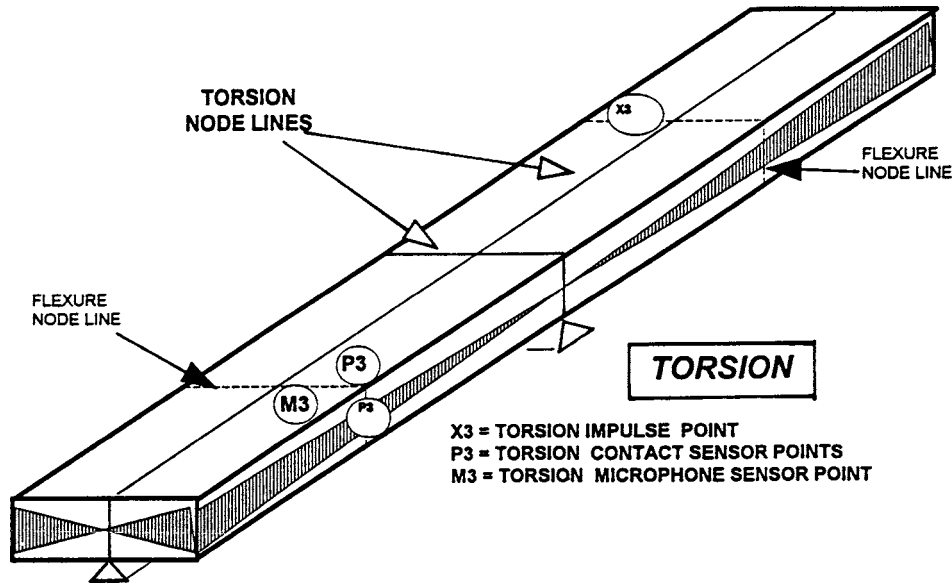10.1.1 For the fundamental flexure frequency of a rectangular bar (2),

FIG. 4 Rectangular Specimen Tested for Torsional Vibration

$$E = 0.9465(mf_f^2 / b)(L^3 / t^3) \, T_1 \qquad (2)$$

where:

E = Young's modulus, Pa,
m = mass of the bar, g (see Note 3),
b = width of the bar, mm (see Notes 2 and 3),
L = length of the bar, mm (see Note 3),
t = thickness of the bar, mm (see Notes 2 and 3),
f_f = fundamental resonant frequency of bar in flexure, Hz, and
T_1 = correction factor for fundamental flexural mode to account for finite thickness of bar, Poisson's ratio, etc.

$$T_1 = 1 + 6.585\,(1 + 0.0752\,\mu + 0.8109\,\mu^2)\,(t/L)^2 - 0.868\,(t/L)^4$$
$$- \left[\frac{8.340\,(1 + 0.2023\,\mu + 2.173\,\mu^2)(t/L)^4}{1.000 + 6.338\,(1 + 0.1408\,\mu + 1.536\,\mu^2)\,(t/L)^2} \right] \qquad (3)$$

$$\mu = Poisson's \;\; ratio$$

NOTE 2—The width (b) and thickness (t) values used in the modulus calculations (Eq 2 and Eq 3) for the rectangular specimens will depend on the type of vibration (out-of-plane or in-plane) induced in the specimen. The cross-sectional dimension t will always be parallel to the vibrational motion. The dimension b will always be perpendicular to the vibrational motion. In effect, the two different flexural modes will give two different fundamental resonant frequencies, but the calculations for the two modes should give the same modulus value, because the values for b and t are exchanged in the calculations for the two different flexure modes.

NOTE 3—In the modulus equations, the mass and length terms are given in units of grams and millimetres. However, the defined equations can also be used with mass and length terms in units of kilograms and metres with no changes in terms or exponents.

10.1.1.1 If $L/t \geq 20$, T_1 can be simplified to the following:

$$T_1 = [1.000 + 6.585\,(t/L)^2] \qquad (4)$$

and E can be calculated directly.

10.1.1.2 If $L/t < 20$ and Poisson's ratio is known, then T_1 can be calculated directly from Eq 3 and then used to calculate E.

10.1.1.3 If $L/t < 20$ and Poisson's ratio is not known, then an initial Poisson's ratio must be assumed to begin the computations. An iterative process is then used to determine a value of Poisson's ratio, based on experimental Young's modulus and shear modulus. The iterative process is flow-charted in Fig. 5 and described in paragraphs (1) through (5) below.

(1) Determine the fundamental flexural and torsional resonant frequency of the rectangular test specimen, as described in Section 9. Using Eq 8 and Eq 9, calculate the dynamic shear modulus of the test specimen for the fundamental torsional resonant frequency.

(2) Using Eq 2 and Eq 3, calculate the dynamic Young's modulus of the rectangular test specimen from the fundamental flexural resonant frequency, dimensions and mass of the specimen, and initial/iterative Poisson's ratio. Care shall be exercised in using consistent units for all of the parameters throughout the computations.

(3) Substitute the dynamic shear modulus and Young's modulus values calculated in steps (1) and (2) into Eq 11 for Poisson's ratio satisfying isotropic conditions. Calculate a new value for Poisson's ratio for another iteration beginning at Step (2).

(4) Repeat Steps (2) and (3) until no significant difference (2 % or less) is observed between the last iterative value and the final computed value of the Poisson's ratio.

(5) Self-consistent values for the moduli are thus obtained.

10.1.1.4 If the rectangular specimen is narrow and the four long edges of the rectangular bar have been chamfered or rounded, then the calculated Young's modulus, E, should be corrected in accordance with Annex A2.
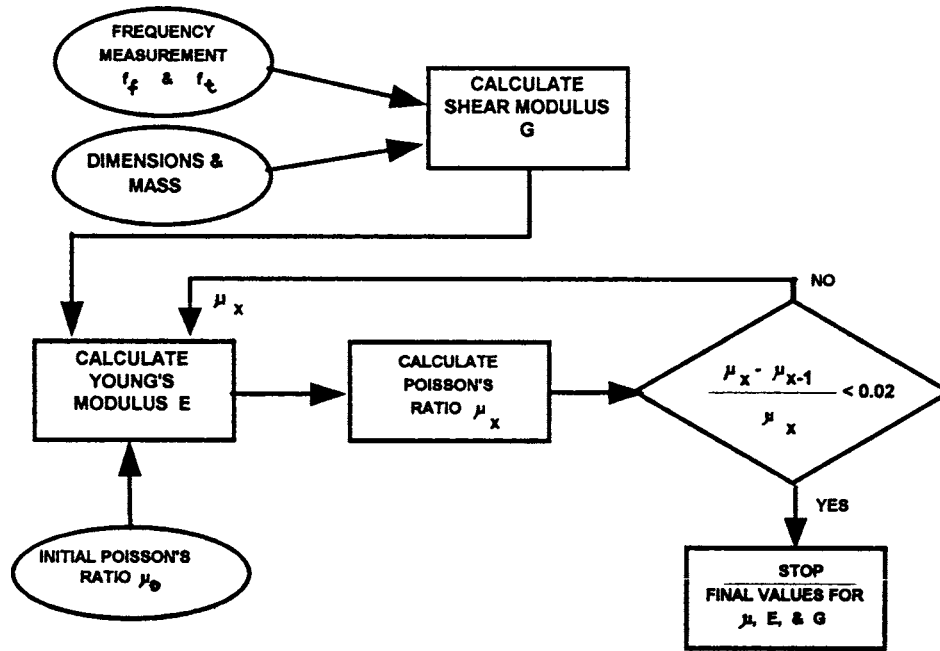
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FIG. 5 Process Flowchart for Iterative Determination of Poisson's Ratio

10.1.2 For the fundamental flexural frequency of a rod of circular cross section **(2)**:

$$E = 1.6067 \, (L^3 / D^4)(mf_f^2)T_1' \tag{5}$$

where:

D = diameter of rod, mm (see Note 2), and
T_1' = correction factor for fundamental flexural mode to account for finite diameter of bar, Poisson's ratio, etc.

$$T_1' = 1 + 4.939 \, (1 + 0.0752 \, \mu + 0.8109 \, \mu^2) \, (D/L)^2$$
$$-0.4883 \, (D/L)^4$$
$$- \left[\frac{4.691 \, (1 + 0.2023 \, \mu + 2.173 \, \mu^2) \, (D/L)^4}{1.000 + 4.754 \, (1 + 0.1408 \, \mu + 1.536 \, \mu^2)(\, D/L)^2} \right] \tag{6}$$

10.1.2.1 If $L / D \geq 20$, then T_1' can be simplified to the following:

$$T_1' = [1.000 + 4.939 \, (D/L)^2] \tag{7}$$

10.1.2.2 If $L / D < 20$ and Poisson's ratio is known, then T_1' can be calculated directly from Eq 6 and then used to calculate E.

10.1.2.3 If $L / D < 20$ and Poisson'a ratio is not known, then an initial Poisson's ratio must be assumed to start the computations. Final values for Poisson's ratio, dynamic Young's modulus, and dynamic shear modulus are determined, using the same method shown in Fig. 5 and described in paragraphs (*1*) through (*5*) in 10.1.1.3, but using the modulus equations for circular bars (Eq 5, Eq 6, and Eq 10).

10.2 *Dynamic Shear Modulus (3,4)*:

10.2.1 For the fundamental torsional frequency of a rectangular bar. **(3)**:

$$G = \frac{4 \, L \, m \, f_t^{\,2}}{b \, t} R \tag{8}$$

where:

G = dynamic shear modulus, Pa,
f_t = fundamental resonant frequency of bar in torsion, Hz,

$$R = \left[\frac{1 + \left(\frac{b}{t} \right)^2}{4 - 2.521 \, \frac{t}{b} \left(1 - \frac{1.991}{e^{\pi \frac{b}{t}} + 1} \right)} \right] \left[1 \right.$$
$$\left. + \frac{0.00851 \, b^2}{L^2} \right] - 0.060 \, \left(\frac{b}{L} \right)^{\frac{3}{2}} \left(\frac{b}{t} - 1 \right)^2 \tag{9}$$

Equation 9 should be accurate to within ~ 0.2 % for b/L $\leq$ 0.3 and b/t $\leq$ 10 in the fundamental mode of torsional vibration, otherwise the errors are estimated to be $\leq$ 1 %. **(3)**

10.2.2 For the fundamental torsion frequency of a cylindrical rod **(3)**:

$$G = 16 \, mf_t^2 (L/\pi D^2) \tag{10}$$

10.3 *Poisson's Ratio:*

$$\mu = (E/2G) - 1 \tag{11}$$

where:

μ = Poisson's ratio,
E = Young's modulus, and
G = shear modulus.

10.4 If measurements are made at elevated or cryogenic temperatures, the calculated moduli must be corrected for thermal expansion effects using Eq 12.

$$M_T = M_0 \, [f_T/f_0]^2 \, [1/(1 + \alpha \Delta T)] \tag{12}$$

M_T = modulus at temperature T (either Young's modulus E or shear modulus G),
M_0 = modulus at room temperature (either Young's modulus E or shear modulus G),

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f_T = resonant frequency in furnace or cryogenic chamber at temperature T,

f_0 = resonant frequency at room temperature in furnace or cryogenic chamber,

α = average linear thermal expansion (mm/mm · °C) from room temperature to test temperature (Test Method C 372 is recommended), and

ΔT = temperature differential in °C between test temperature T and room temperature.

10.5 Use the following stress conversion factor for English units.

$$1 \text{ Pa} = 1.450 \times 10^{-4} \text{ psi} \qquad (13)$$

11. Report

11.1 Report the following information:

11.1.1 Identification of specific tests performed, a detailed description of apparatus used (impulser, transducer, electrical system, and support system), and an explanation of any deviations from the described test method.

11.1.2 Complete description of material(s) tested stating composition, number of specimens, specimen geometry and mass, specimen history, and any treatments to which the specimens have been subjected. Include comments on dimensional variability, surface finish, edge conditions, observed changes after cryogenic or high-temperature testing, etc., where pertinent.

11.1.3 For each specimen tested – the specimen geometry, dimensions and mass, specimen test temperature, the number of measurements taken, the individual numerical values and the mean value obtained for the measured fundamental resonant frequency for each test mode, and the calculated values for dynamic Young's modulus, dynamic shear modulus, and Poisson's ratio.

11.1.4 Date of test and name of the person performing the test.

11.1.5 Laboratory notebook number and page on which test data are recorded or the computer data file name, or both, if used.

12. Precision and Bias

12.1 An evaluation (6) was conducted and published in 1990, by Smith, Wyrick, and Poole, of three different methods of modulus measurement of mechanically alloyed materials. As part of that evaluation, the impulse modulus measurement method,[4] using a commercial instrument, was used. With that instrument, the precision of the impulse method was measured using a NIST Standard Reference Material 718 (alumina reference bar No. C1) in flexural vibration. The NIST standard had a measured and specified fundamental flexural frequency of 2043.3 Hz. The fundamental flexural resonant frequency of the NIST reference bar was measured by the impulse method and reported by Smith, Wyrick, and Poole as 2044.6 Hz. This

TABLE 1 Within- and Between-Laboratory Precision

Test Bar No. and Type	Al_2O_3	Si_3N_4
Measured fundamental flexural frequency (Hz)	6581	11.598
95 % repeatability limit (within laboratory) 2.8 CV, %$_r$[A]	0.8%	1.1%
95 % reproducibility limit (between laboratories) 2.8 CV, %$_R$[A]	1.2%	1.4%

[A] Calculated in accordance with Practice E 691, Section 21, and reported in accordance with Practice E 177, Section 28.

was a percentage error of +0.06 %, indicating the level of bias that is achievable with the impulse method.

12.2 An intralaboratory round-robin test was conducted in 1993 to measure the precision of frequency measurement on two monolithic ceramic test bars. A bias test was not conducted because suitable standard reference bars were not readily available.

12.2.1 The tests were conducted with an alumina test bar (10 g, 83.0 by 6.9 by 4.8 mm) and a silicon nitride bar (2.0 g, 50 by 4.0 by 3.0 mm). The silicon nitride bar was machined to Test Method C 1161 tolerances; the alumina bar was not machined and varied from 4.5 to 4.8 mm in thickness along its length. The variations in the alumina bar thickness were deliberate; it provided a test of the robustness of the frequency measurement technique.

12.2.2 Torsional frequency measurements were not performed because the width-thickness ratio of the bars was not suitable for torsional frequency measurements.

12.2.3 The bars were tested in flexural vibration at eight laboratories using ten combinations of different frequency analyzer test systems, impulsers, contact and non-contact transducers, and supports systems. For the alumina bar, the mean measured flexural frequency for the ten tests was 6581 Hz, with a standard deviation of 20 Hz. This corresponds to a coefficient of variation of 0.3 %. For the silicon nitride bar, the mean measured flexural frequency for the ten tests was 11 598 Hz, with a standard deviation of 34 Hz. This corresponds to a coefficient of variation of 0.3 %.

12.2.4 The intralaboratory study did show that individuals with experience in using the impulse test method for a given specimen geometry produced data sets with smaller standard deviations. For example, with the alumina test bar, the coefficients of variation for individual laboratories ranged from 0.001 to 0.6 % among the ten test sets. For the silicon nitride bar, the range of coefficients of variation was 0.001 to 1.0 % for the individual laboratories.

12.2.5 Based on this intralaboratory study of the impulse test method, the repeatability and reproducibility coefficients at the 95 % confidence level are listed in Table 1.

12.3 A propagation of errors analysis of the equations for E and G using the stated tolerances for dimensions, mass, and frequency measurements in this test method has shown that a 0.1 % error in the measurement of the key variables produces a range of errors in the calculation of the modulus based on the variable exponent in the equations. Table 2 gives the calculation error effects of errors in the different experimental variables.

12.4 It is expected that the major sources of experimental variation in modulus values for this test method will be in two measurements—the fundamental frequency and the smallest

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[4] Grindosonic instrument, available from J. W. Lemmens, 10801 Pear Tree Lane, St. Louis, MO 63074.

TABLE 2 Effects of Variable Error on Modulus Calculation

Variable	Measurement Error	Variable Exponent in Modulus Equation	Calculation Error
Frequency (f)	0.1%	f^2	0.2%
Length (L)	0.1%	L^3	0.3%
Mass (m)	0.1%	m	0.1%
Width (b)	0.1%	b^{-1}	0.1%
Thickness (t)	0.1%	t^{-3}	0.3%
Diameter (D)	0.1%	D^{-4}	0.4%

dimension (thickness/diameter) of the test bars. If a fundamental resonant frequency of 6000 Hz is measurable to an accuracy of 18 Hz/(0.3 %) and a 3-mm thick bar is parallel and measured to an accuracy of 0.01 mm (0.3 %), the error in the thickness measurement will have the greater effect on the modulus calculation (0.9 % for thickness error versus 0.6 % for frequency error).

13. Keywords

13.1 advanced ceramics; bar; beam; cylindrical rod; disc; dynamic; elastic modulus; flexure; elastic properties; impulse; Poisson's ratio; resonance; resonant beam; shear modulus; torsion; Young's modulus

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ANNEXES

(Mandatory Information)

A1. DISC-SHAPED SPECIMENS FOR DYNAMIC YOUNG'S MODULUS, SHEAR MODULUS, AND POISSON'S RATIO FOR ADVANCED CERAMICS BY IMPULSE EXCITATION OF VIBRATION

A1.1 Scope

A1.1.1 In testing advanced ceramic disc specimens for Young's modulus, shear modulus, and Poisson's ratio, the disc geometry requires a significantly different set of equations and method of calculation and some minor changes in procedures. This annex describes those variations as they relate to terminology, test specimens, procedures and calculations for the disc geometry specimens. However, the general methods, references, terminology, significance, interferences, apparatus, specimen requirements, and procedures described in the main body of the test method are still pertinent to the test procedure and the results.

A1.2 Terminology

A1.2.1 *Description of Terms Specific to this Annex for Disc-Shaped Specimens:*

A1.2.1.1 *antinodes*, *n*—two or more locations that have local maximum displacements in an unconstrained resonant specimen.

A1.2.1.2 *first natural vibration*, *n*—the vibrations that occur when the displacements in the cross-sectional plane (the plane that is parallel to the flat of the disc) are normal to the plane and symmetrical around two orthogonal diameters in the plane of the disc, producing a twisting of the disc. This is an orthogonal anti-flexural mode of vibration (7). For the first natural vibration mode, the nodes are located along two orthogonal diameters, offset 45° from the point where the vibration was induced. The antinodes are located along two orthogonal (90° offset) diameters in the disc, with one diameter intersecting the point where the vibration was induced. See Fig. A1.1, which shows a finite element map of anti-flexural displacement in a disc and a schematic of the nodal and antinodal lines for the first natural vibration of the disc).

A1.2.1.3 *nodes*, *n*—one or more locations that have a constant zero displacement in an unconstrained resonant specimen.

A1.2.1.4 *second natural vibration*, *n*—the vibrations that occur when the displacements in the cross-sectional plane (the plane that is parallel to the flat of the disc) are normal to the plane and are uniform in displacement for a given radial distance from the center point through the entire 360° arc. This is an axisymmetric flexural vibration (7). For the second natural vibration mode of a disc, the nodes are located in a circle concentric with the center of the disc with a fractional radius of 0.681 of the disc radius. The antinodes are located at the center and around the circumference of the disc specimen. See Fig. A1.2, which shows a finite element map of axisymmetric flexural displacement in a disc and a schematic of the nodal circle and the antinodal points and line for the second natural vibration of the disc.

A1.3 Test Specimen

A1.3.1 Fabricate the specimens so that they are disc-shaped with a diameter to thickness ratio of at least 4, with a value of 10-20 recommended for experimental simplicity. Resonant frequencies for a given specimen are a function of the specimen dimensions as well as its mass and moduli; therefore, dimensions should be selected with this relationship in mind. By using approximate specimen dimensions and estimated values for the moduli and Poisson's ratio, nominal resonant frequencies can be calculated with the formulas in A1.5. By adjusting the size of the specimen, the resonant frequencies can be "tuned" into the measurement range of the transducers and electronics. The dimensional tolerances for the thickness, diameter, and flatness are given in A1.3.2-A1.3.5.

A1.3.2 Measure the specimen thickness within 0.002 mm or 0.1 % (whichever is greater) at five locations (one at the center, two at the outer edge, and two at the ½ radius). Determine the average of the five measurements for a specimen thickness.

A1.3.3 Ensure that the thickness of the disc is uniform so that opposite plane surfaces of the disc are parallel to within 0.002 mm or ± 0.1 % of the thickness, whichever is greater. For larger discs where precision machining of the thickness to those tolerances is difficult, an alternative tolerance of 0.01 mm or ± 0.5 % is allowed, with proper allowances for the resulting
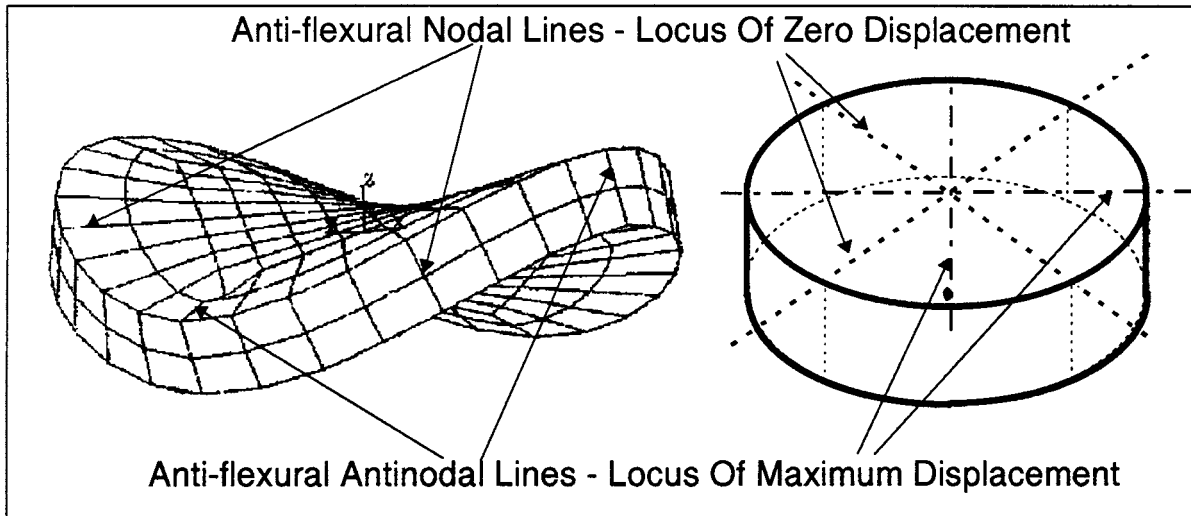
10

FIG. A1.1 Displacement Diagram for Disc-Shaped Specimen in First Natural Vibration
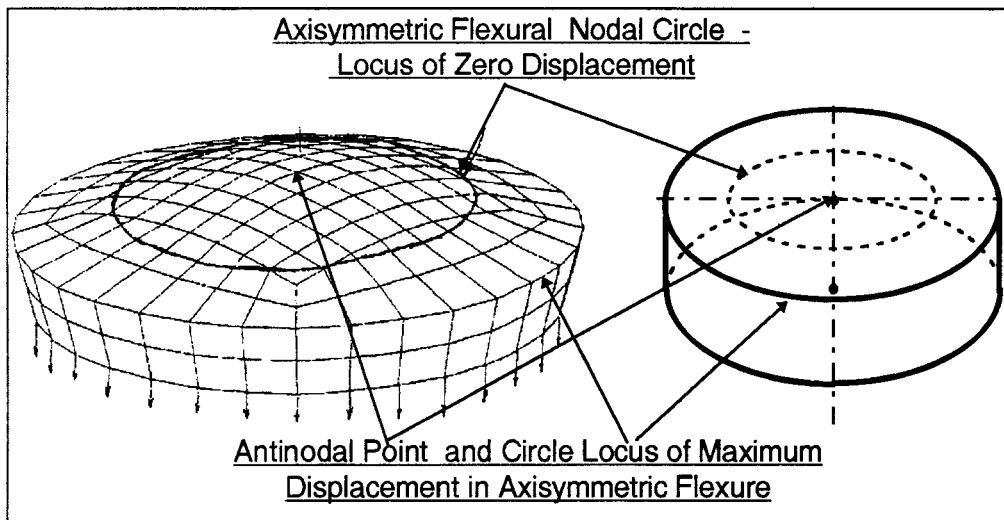

FIG. A1.2 Displacement Diagram for Disc-Shaped Specimen in Second Natural Vibration Mode

loss of precision in the calculated modulus. The use of the less precise tolerance for the thickness should be noted in the report.

A1.3.4 Measure the specimen diameter within 0.01 mm or 0.1 % at four locations (45° intervals around the circumference, as shown in Fig. A1.3.) Determine an average from the four measurements. Ensure that the diameter of the disc is uniform to within 0.01 mm or ± 0.1 %, whichever is greater. (The value of the radius is used to determine the Poisson's ratio.)

A1.3.5 Measure the flatness of the disc resting on an surface plate with a dial gauge, taking measurements at nine locations (one at the center, four at the outer edge, and four at the ½ radius) on the disc. Turn the disc over and repeat the measurement on the opposite face. The maximum difference between any two measurements on a face must not exceed 0.1 % of the diameter.

A1.3.6 Determine the specimen mass to the nearest 10 mg or 0.1 % of the total weight, whichever is greater.
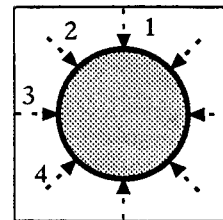

FIG. A1.3 Diameter Measurement Locations

A1.3.7 All other specimen requirements and recommendations as described in Section 8 of the main body of this standard are pertinent to the disc-shaped specimens.

A1.4 Procedure

A1.4.1 Activate and allow all electrical equipment to stabilize according to manufacturer's recommendations.

A1.4.2 Use a test specimen established as an "in-house" verification standard to check the operation of the test system. The in-house standard can also be used to verify the operation,
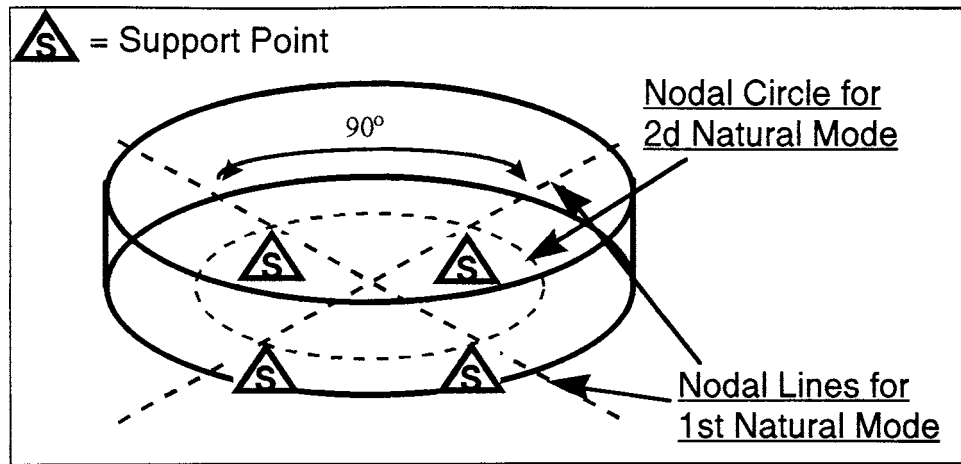
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FIG. A1.4 Supports Points for 1ˢᵗ and 2ᵈ Natural Vibrations in Discs

repeatability and precision of the system and the operator. The standard specimen should have a geometry similar in size and shape to the experimental specimens.

A1.4.3 *First Natural Resonant Frequency:*

A1.4.3.1 Specimens can be supported either on soft polyurethane foam strips or on four hard support points. Specimens with a low (< 10) diameter-to-thickness ratio may be supported on flat strips of soft polyurethane foam set in an x-pattern. Experience has shown that foam supports are more compliant than hard supports and markedly reduce rattling and extraneous vibrations in the test. If foam supports are not available, place the specimen on four hard support points located at the intersections of the nodal diameters of the first natural vibration with the nodal circle of the second natural vibration (See Fig. A1.4). Generally, visual positioning is sufficiently accurate for larger specimens (diameter > 75 mm). Specimens with a high (> 20) diameter-to-thickness ratio or a smaller diameter (< 75 mm) will require more accurate positioning.

NOTE A1.1—For precise support location, the nodal circle diameter of the test specimen can be calculated as 0.681 of the geometrical mean diameter. Then draw, or otherwise define, the nodal circle of this diameter on a piece of paper so that it is concentric with a circle drawn slightly larger than the diameter of the test piece specimen. Locate four equally disposed positions around the nodal circle to define the support points (see Fig. A1.4). Place the supports at these positions and then place the test specimen on the supports so that the specimen is visually concentric with the second larger drawn circle.

A1.4.4 Follow 9.3.2 in the main body for orienting and using contact and non-contact transducers. Determine the direction of maximum sensitivity for the transducer. Orient the transducer so that it will detect the desired vibration.

A1.4.5 For the first natural vibration Fig. A1.5 shows the impulse and sensor points. Locate the transducer on a point (S1 in Fig. A1.5) on a second mode nodal circle on the flat of the disc and offset by 45° from a support point. Strike the specimen on the flat of the disc (X1 in Fig. A1.5) 90° away from the transducer point. Striking at the second mode nodal circle will minimize the possibility of exciting that particular mode of vibration.

NOTE A1.2—Accuracy and repeatability in the impulse excitation test depend upon developing and detecting a single vibration mode in the test

specimen, without introducing and detecting alternative vibration modes in the specimen. Exciting and detecting a singular vibration mode for a simple geometry of suitable size depends on three experimental variables:

— the alignment of the support points with the specimen nodal points to favor the desired vibration mode and to minimize interference
— the location of the strike point at the antinode of the desired vibration mode and an elastic (rapid with short duration contact) method of striking
— the positioning of the detection transducer at a point where the desired vibration mode is active, but where there is minimal interference from competitive vibrations

With a minimal amount of experience and practice on a suitable specimen geometry, most operators can obtain repeatability and accuracy on the order of 1 % or better in a series of repeated test strikes. For a specific specimen, the first test strikes are a means of determining the appropriate locations and methods of specimen support, striking, and signal pick-up. By slight shifts in those positions and repeated test strikes, the proficient operator can find the appropriate locations to achieve the consistent 1 % repeatability in a series of five consecutive readings. This is an iterative process for every new specimen; but can be done quickly by those operators who have experience with specimens of different sizes and compositions.

A1.4.6 After repeatability of the test procedure has been demonstrated, take and record at least five (5) readings. Use the average of these five readings to determine the first natural frequency (f_1). If readings repeat with a variation greater than 1 %, the operator shall review the measurement technique (supports, tap/sensor location, tapping mode) and the integrity/geometry of the test specimen.

NOTE A1.3—Adjustments and improved consistency in technique may improve the repeatability. Measurement technique can also be checked for consistency by taking measurements on "in-house" calibration standards. If technique adjustments do not improve the repeatability for the experimental specimens, the operator should determine what the source of the variation is. There are two possible causes for this variation:

— a geometry in which two vibration modes exist with similar frequencies that interfere during the measurement.
— inhomogeneities/flaws in the test specimens which produce spurious vibrations or very rapid damping.

In the case of specimens with vibration modes that are similar in frequency, a frequency spectrum analyzer (which displays multiple frequencies) is particularly useful in isolating the desired vibration mode. With the analyzer showing the competitive vibration frequencies, the harmonic vibration of interest can be enhanced and the undesirable vibration can be diminished, based on iterative changes in technique (support/ tap/sensor location, tapping mode).
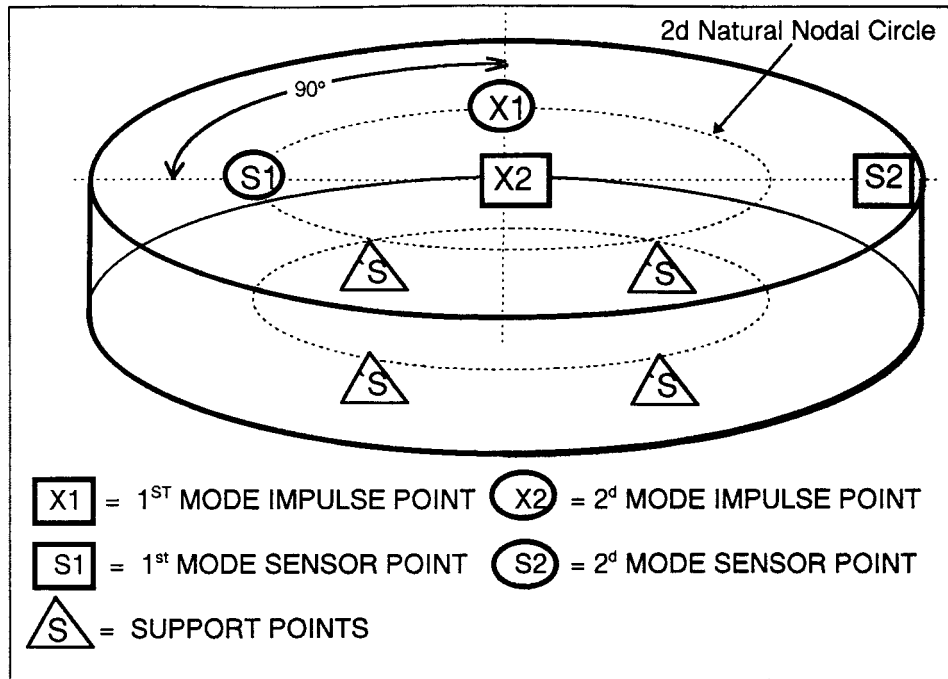
12

FIG. A1.5 Impulse and Sensor Points for 1st and 2d Natural Vibration in Discs

A1.4.7 *Second Natural Resonant Frequency*:

A1.4.7.1 The specimen is supported at the same points used for the first natural frequency support–the intersection of the first natural frequency nodal diameters with the second natural frequency nodal circle (Fig. A1.4).

A1.4.7.2 Position the transducer on the flat of the disc close to the outer circumference. (S2 in Fig. A1.5). Then strike the disc specimen lightly at the center of the specimen (X2 in Fig. A1.5).

A1.4.7.3 After repeatability of the test procedure has been demonstrated, the operator takes at least five (5) readings. Use the average of these five readings to determine the second natural frequency (f_2). If the readings vary by more than 1 % of each other, the operator shall review the measurement technique. [See Note A1.2 and Note A1.3 following sections A1.4.5 and A1.4.6]

A1.5 Calculations

A1.5.1 The derivation and use of the equations for calculating the Poisson's ratio and moduli from disc-shaped specimens are described in detail in two references (**8 and 9**). Reference (**8**) gives the derivation and procedures for the baseline calculation. The fundamental equation defining the relationship between the natural resonant frequency, the material properties, and the specimen dimensions is given in (**8**) as:

$$f_1 = \frac{K_1}{2\pi r^2} \sqrt{\frac{A}{\rho t}} \qquad (A1.1)$$

where f_1 is the resonant frequency of interest, K_1 is the geometric factor for that resonant frequency, r is the radius of the disc, A is the plate constant ($A = Et^3/[12 (1- \mu^2)]$), t is the disc thickness, ρ is the density of the disc, E is Young's modulus of elasticity, and μ is the Poisson's ratio for the disc material. This is a general equation which is valid for both the first natural and second natural vibrations. Ref (**9**) supplements the (**8**) with more extensive tables for the geometric factors K_i and for determining Poisson's ratio. The overall method for calculating the Poisson's ratio, Young's modulus, and the shear modulus from the first natural and second natural frequencies is described in (**8**) and (**9**) and as the following three-step procedure.

(1) Determine the Poisson's ratio from the experimental values for the first and second natural resonant frequencies.

(2) Calculate two independent values for E (Young's modulus of elasticity) using the Poisson's ratio from step 1 and the first natural and second natural resonant frequencies. Determine E as the average of the two independent calculations.

(3) Calculate the value of G (Shear Modulus) using the Poisson's ratio from Step 1 and the calculated value of E from step 2.

The details for each calculation are given in the following sections.

A1.5.2 *Poisson's Ratio* Refs. (**8 ,9, 10**)—For the disc-shaped specimen, Poisson's ratio (μ) can be determined directly from the experimental values of the first natural resonant frequency (f_1) and the second natural resonant frequency (f_2). This is done by using Eq A1.2 which uses the ratio (f_2/f_1) of the second natural resonant frequency to the first natural resonant frequency, the ratio (t/r) of the specimen thickness to the specimen radius, and the polynomial coefficients from Table A1.1.

$$\mu = a + b\,(t/r) + c\,(f_2/f_1)^{-1} + d(t/r)^2 + e(f_2/f_1)^{-2} + f(t/r)(f_2/f_1)^{-1}$$
$$+ g(t/r)^{+3} + h(f_2/f_1)^{+3} + i(t/r)(f_2/f_1)^{-2} + j(t/r)^2 (f_2/f_1)^{-1} \qquad (A1.2)$$

NOTE A1.4—The fits with the polynomial equation are within 1 % of the data tables in Glandus (9). For the wider range of $0.00 \leq t/R \leq 0.500$, and $1.400 \leq f_2/f_1 \leq 1.900$, μ can be estimated within 1.3 % of the data of

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TABLE A1.1 Constants for Estimation of μ for Eq A1.2

Coefficient	f_2/f_1 range			
	1.375 – 1.525	1.525 – 1.725	1.725 – 1.900	1.400 – 1.900
a	0.7161	− 4.3343	− 8.0933	− 4.30526
b	− 0.6502	2.1017	3.4452	0.38999
c	− 4.1377	5.6718	12.2471	6.10124
d	1.0459	0.5107	− 2.2461	0.19582
e	4.2095	− 2.2025	− 6.1198	− 2.76353
f	0.6056	− 2.5965	− 3.2070	− 0.47553
g	− 0.5148	− 0.5109	− 0.0872	− 0.62008
h	− 1.1313	0.2792	1.0725	0.45518
i	− 0.1247	0.7924	0.7117	0.11816
j	− 0.3197	0.0283	1.4525	0.31933

Glandus (9) by using the constants in the last column of Table A1.1 with Eq A1.2. The coefficient values corresponding to f_2/f_1= 1.350 were not included in the fit because they decreased the quality of fit and are not commonly encountered. [From Salem (10)]

A1.5.3 *Dynamic Young's Modulus* **(8, 9, 10)**

A1.5.3.1 For the Young's modulus of a disc, two calculations of E (E_1 and E_2) are made independently from the two resonant frequency measurements, and then a final value E is determined by averaging the two calculated values E_1 and E_2.

$$E_1 = [37.6991\, f_1^2\, D^2\, m\, (1-\mu^2)]\,/\,(K_1^2\, t^3) \qquad (A1.3)$$

$$E_2 = [37.6991\, f_2^2\, D^2\, m\, (1-\mu^2)]\,/\,(K_2^2\, t^3) \qquad (A1.4)$$

$$E = (E_1 + E_2)\,/2 \qquad (A1.5)$$

where:
E = Young's Modulus (Pa)
E_1 = First Natural Calculation of Young's Modulus
E_2 = Second Natural Calculation of Young's Modulus
f_1 = First Natural Resonant Frequency (Hz) of the disc
f_2 = Second Natural Resonant Frequency (Hz) of the disc
D = Diameter (mm) of the disc
m = Mass (g) of the disc
μ = Poisson's ratio for the specimen as determined in A1.5.2
K_1 = First Natural Geometric Factor from Eqs A1.6 or A1.7 and Table A1.2
K_2 = Second Natural Geometric Factor from Eqs A1.6 or A1.7 and Table A1.2
t = thickness (mm) of the disc
r = radius (mm) of the disc

For $0 \le t/r \le 0.500$ and $0 \le \mu \le 0.500$, K_1 and K_2 can be determined by Eq A1.6 and the coefficients from Table A1.2

$$K_i = \frac{(a + b(t/r) + c(t/r)^2 + d\mu + e\mu^2 + f\mu^3)}{(1 + g(t/r) + h(t/r)^2 + i\mu + j\mu^2)} \qquad (A1.6)$$

TABLE A1.2 Polynomial Constants for Eqs A1.6 and A1.7

Coefficient	$0 \le t/r \le 0.500$ and $0 \le \mu \le 0.500$ Eq A1.6		$0.100 \le t/r \le 0.200$ and $0.14 \le \mu \le 0.34$ Eq A1.7	
	K_1	K_2	K_1	K_2
a	6.1528	8.2584	6.2305	8.3901
b	0.5085	−2.3794	−1.7997	−2.9434
c	0.8825	6.6164	−2.4825	2.0928
d	−3.9037	2.3540	3.6354	5.0895
e	−5.4739	−9.1109	−0.3022	1.9810
f	3.9297	−0.9923	3.6488	2.7693
g	0.1364	−0.2142	−9.3417	−18.3125
h	0.9120	1.7467	−2.2626	−3.0307
i	−0.3628	−0.0475	−3.3588	−3.1336
j	−0.3612	−0.9262	−10.7968	−15.1250

The fits for K_1 and K_2 for Eq A1.6 are within 0.5% of the data tables from Glandus (9). [From Salem 10].

For the narrow range of $0.100 \le t/r \le 0.200$ and $0.14 \le \mu \le 0.34$, K_1 and K_2 can be determined by Eq A1.7 and the coefficients from Table A1.2.

$$K_i = a + b(t/r) + c\mu + d(t/r)^2 + e\mu^2 + f(t/r)\mu + g(t/r)^3 + h\mu^3 + i(t/r)\mu_2 + j(t/r)_2\,\mu \qquad (A1.7)$$

The fits for K_1 and K_2 for Eq A1.7 are within 0.1 % of the data tables from Glandus (9). [From Salem 10].

Note A1.5—The constant 37.6991 in the equation is from **(9)** and is the numerical value of 12π.

A1.5.4 *Dynamic Shear Modulus (9)*

A1.5.4.1 The shear modulus is determined from the calculated Young's modulus value and the Poisson ratio.

$$G = E\,/\,[2(1 + \mu)] \qquad (A1.8)$$

where:
G = Shear Modulus (Pa)
E = Young's Modulus (Pa) calculated in section A1.5.3
μ = Poisson's ratio determined in section A1.5.2

A2. CORRECTION FOR EDGE CHAMFERS OR RADII IN RECTANGULAR BEAMS IN THE CALCULATION OF YOUNG'S MODULUS

A2.1 *Introduction*

A2.1.1 This modulus standard uses a rectangular specimen with a simple prismatic cross section for calculating the dynamic Young's modulus using Eq 2. In actual practice, rectangular specimens with edge chamfers or radii, as illus-

trated in Figs. A2.1 and A2.2, are frequently used for mechanical testing. (The edge treatment is used with flexure strength specimens to reduce or eliminate the sensitivity to edge damage). The modulus equation (Eq 2) in the standard does not account for the effect of such edge treatments on the moment
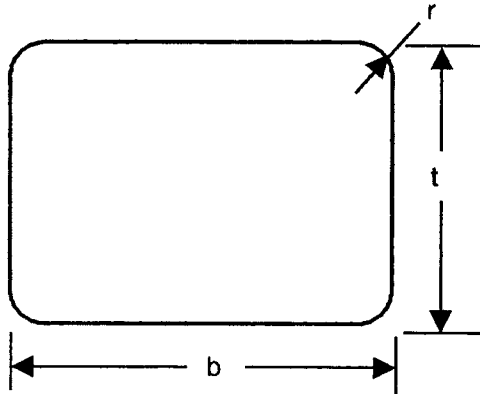
14

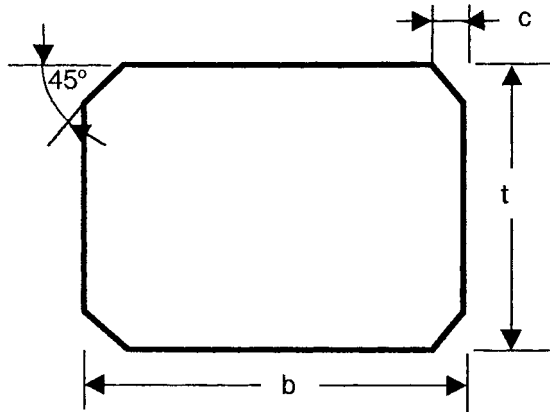**FIG. A2.1 Specimen Cross Section for a Rounded-Edge Beam**

FIG. A2.2 Specimen Cross Section for a Chamfered-Edge Beam

of inertia and the density, and subsequent effects on the dynamic Young's modulus.

A2.1.2 This annex provides a simple means to modify Eq 2 to correct the calculated Young's modulus for the two types of edge treatments. This analysis and corroborative experimental data are from reference (11). The corrections to E are significant (0.5 % or greater) for narrow specimens which are typical of flexure strength test configurations, (for example C 1161). The corrections are less significant for wide specimens (w/t >5) such as those recommended in 8.2. These adjustments are only applicable for flexural modes of resonance and are not appropriate for the longitudinal resonance mode or for torsional resonance.

A2.2 *Measurement Procedure*

A2.2.1 Measure the chamfer size, *c*, or the rounded edges, *r*, of the rectangular specimen by any convenient method to the same accuracy used for the overall dimensions. A traversing stage under a microscope, a traveling microscope, or an optical comparator may be suitable. Use the average *c*, or *r*, for the correction. The correction factors and equations below may be less accurate if the chamfers or rounded edges are uneven or dissimilar in size. The correction factors only applicable if all four long edges are treated.

A2.3 *Moment of Inertia Correction*

A2.3.1 The true Young's modulus, E_cor, for symmetrically chamfered specimens may be calculated as follows:

$$E_{cor} = \left(\frac{I_b}{I_t}\right) E_b \qquad (A2.1)$$

where E_b and I_b are the calculated Young's modulus and moment of inertia assuming the beam is a simple rectangular beam, uncorrected for chamfers or rounds, respectively. I_t is the true moment of inertia of a beam with four symmetric chamfers or edge radii applied to the long edges of the beam.

A2.3.2 Chamfers reduce the moment of inertia, *I*, and slightly alter the radius of gyration. The effect upon *I* previously has been quantified in connection with work to minimize experimental error in flexure strength testing (Refs. 12, 13, 14). Even a small chamfer can alter *I* a meaningful amount. For example, a 45° chamfer of 0.15 mm size will reduce I by 1 % for common 3 mm × 4 mm ceramic flexure strength specimens. The moment of inertia, I_b, for a rectangular cross section beam of thickness, t, and width, b, (with no chamfer) is:

$$I_b = \frac{bt^3}{12} \qquad (A2.2)$$

A2.3.3 The true moment of inertia, I_t, for a beam with four 45° chamfers of size c along the long edges is (Refs. 12, 13):

$$I_t = \frac{bt^3}{12} - \frac{c^2}{9}\left(c^2 + \frac{1}{2}(3t - 2c)^2\right) \qquad (A2.3)$$

where the second term on the right hand side shows the reduction due to the chamfers. It is assumed that the four chamfers are identical in size.

A2.3.4 The true moment of inertia, I_t, for a beam with four identical rounded edges of radius r is (Ref. 13)

$$I_t = \frac{b(t-2r)^3}{12} + \frac{(b-2r)r^3}{6} + \frac{(b-2r)(t-r)^2 r}{2} + 4r^4\left(\frac{\pi}{16} - \frac{4}{9\pi}\right)$$
$$+ \pi r^2\left(\frac{t}{2} - r\left(1 - \frac{4}{3\pi}\right)\right)^2 \qquad (A2.4)$$

The true Young's modulus, E_cor, may be determined from Eq A2.1

A2.3.5 For standard 3 mm × 4 mm rectangular cross section flexure strength specimens (C 1161 size B) Eq A2.1 may be expressed:

$$E_{cor} = FE_b \qquad (A2.5)$$

Correction factors F for a standard 3 mm × 4 mm specimen with four chamfered edges are given in Table A2.1. Analogous values of F for standard 3 mm × 4 mm specimens with four rounded edges, r, are given in Table A2.2

A2.4 *Density Correction*

A2.4.1 An additional correction, but of lesser magnitude, may also be incorporated. Eq 2 in this standard contains an assumption (References 1 and 3) that the density is related to the mass and physical dimensions of the rectangular beam following Eq A2.6:

$$\rho_b = \frac{m}{btL} \qquad (A2.6)$$

A2.4.2 However, edge treatments alter the relationship between the density, mass and physical dimensions of the test piece. If an edge treated beam is used to determine the dynamic Young's modulus, then Eq A2.6 is invalid and an additional correction should be made to E as follows.
The correct density, ρ_t, of a chamfered beam is:

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TABLE A2.1 Correction factors, F and P, for chamfered standard 3mm × 4mm strength test specimens for ASTM C 1161. A chamfer size of 0.150 mm is the maximum value allowed for this geometry by ASTM C 1161 and ISO 14704.

Chamfer Dimension, c (mm)	Moment Correction factor, F b = 4 mm, t = 3 mm	Density Correction factor, P b = 4 mm, t = 3 mm
0.080	1.0031	1.0011
0.090	1.0039	1.0014
0.100	1.0048	1.0017
0.110	1.0058	1.0020
0.115	1.0063	1.0022
0.118	1.0066	1.0023
0.120	1.0069	1.0024
0.122	1.0071	1.0025
0.124	1.0073	1.0026
0.126	1.0076	1.0027
0.128	1.0078	1.0027
0.130	1.0080	1.0028
0.132	1.0083	1.0029
0.134	1.0085	1.0030
0.136	1.0088	1.0031
0.138	1.0090	1.0032
0.140	1.0093	1.0033
0.150	1.0106	1.0038
0.160	1.0121	1.0043
0.170	1.0136	1.0048
0.180	1.0152	1.0054
0.190	1.0169	1.0061
0.200	1.0186	1.0067
0.210	1.0205	1.0074
0.220	1.0224	1.0081
0.230	1.0244	1.0089
0.240	1.0265	1.0097
0.250	1.0287	1.0105

TABLE A2.2 Correction factors, F and P, for edge rounded standard 3mm × 4mm strength test specimens for ASTM C 1161. A rounded edge of 0.200 mm is the maximum value allowed for this geometry by ASTM C 1161 and ISO 14704.

Radius Dimension, r (mm)	Moment Correction factor, F b = 4 mm, t = 3 mm	Density Correction factor, P b = 4 mm, t = 3 mm
0.080	1.0013	1.0005
0.090	1.0017	1.0006
0.100	1.0021	1.0007
0.110	1.0025	1.0009
0.120	1.0030	1.0010
0.130	1.0035	1.0012
0.140	1.0041	1.0014
0.150	1.0046	1.0016
0.160	1.0053	1.0018
0.170	1.0059	1.0021
0.180	1.0066	1.0023
0.190	1.0074	1.0026
0.200	1.0082	1.0029
0.210	1.0090	1.0032
0.220	1.0098	1.0035
0.230	1.0107	1.0038
0.240	1.0116	1.0041
0.250	1.0126	1.0045
0.260	1.0136	1.0049
0.270	1.0146	1.0052
0.280	1.0157	1.0056
0.290	1.0168	1.0061
0.300	1.0180	1.0065

3 mm × 4 mm specimens with either four chamfered or rounded edges are listed in Tables A2.1 and A2.2.

A2.5 *Combined Correction*

A2.5.1 To correct E for both the change in moment of inertia and density due to edge treatments on a standard 3mm × 4 mm cross section specimens:

$$E_{cor} = \left(\frac{I_b}{I_t}\right)\left(\frac{\rho_t}{\rho_b}\right)E_b = FPE_b \qquad (A2.10)$$

NOTE A2.1—This standard calculates E using Eq 2 which requires the beam mass and physical dimensions. Other standards and equipment may use alternative equations that use the material's density (measured by Archimedes water displacement or a similar technique) to calculate E. In such cases, only the moment of inertia correction, Eq A2.5, is necessary.

$$\rho_t = m/[L\,(bt - 2c^2)] \qquad (A2.7)$$

The correct density, ρ_t, for an edge-rounded beam is:

$$\rho_t = m/[L\,(bt - r^2\,(4 - \pi))] \qquad (A2.8)$$

and then:

$$E_{cor} = \left(\frac{\rho_t}{\rho_b}\right)E_b = PE_b \qquad (A2.9)$$

where P is the correction factor for the change in the ρ relationship due to the edge treatment. Values of P for standard

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