# Standard Guide for Determination of the Thermal Resistance of Low-Density Blanket-Type Mineral Fiber Insulation¹ 


#### Abstract

This standard is issued under the fixed designation C 653; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon ( $\epsilon$ ) indicates an editorial change since the last revision or reapproval.


## 1. Scope

1.1 This guide describes the calculation and interpolation of a thermal resistance value for low-density blanket-type insulation material at a particular density and thickness having been selected as representative of the product. It requires measured values of this average density and thickness, as well as apparent thermal conductivity values determined by either Test Method C 177, C 518, or C 1114.
1.2 This guide applies to a density range for mineral-fiber material of roughly 6.4 to $48 \mathrm{~kg} / \mathrm{m}^{3}$ ( 0.4 to $3.0 \mathrm{lb} / \mathrm{ft}^{3}$ ). It is primarily intended to apply to low-density, mineral-fiber mass insulation batts and blankets, exclusive of any membrane facings. Apparent thermal conductivity data for these products are commonly reported at a mean temperature of $23.9^{\circ} \mathrm{C}\left(75^{\circ} \mathrm{F}\right)$ and a hot-to-cold plate temperature difference of $27.8^{\circ} \mathrm{C}\left(50^{\circ} \mathrm{F}\right)$ or $22.2^{\circ} \mathrm{C}\left(40^{\circ} \mathrm{F}\right)$.
1.3 This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory limitations prior to use.

## 2. Referenced Documents

### 2.1 ASTM Standards: ${ }^{2}$

C 167 Test Methods for Thickness and Density of Blanket or Batt Thermal Insulations
C 168 Terminology Relating to Thermal Insulation
C 177 Test Method for Steady-State Heat Flux Measurements and Thermal Transmission Properties by Means of the Guarded-Hot-Plate Apparatus
C 518 Test Method for Steady-State Thermal Transmission Properties by Means of the Heat Flow Meter Apparatus
C 1045 Practice for Calculating Thermal Transmission Properties Under Steady-State Conditions

[^0]
## C 1114 Test Method for Steady-State Thermal Transmission Properties by Means of the Thin-Heater Apparatus

## 3. Terminology

3.1 Definitions-For definitions used in this guide, refer to Terminology C 168.
3.2 Definitions of Terms Specific to This Standard:
3.2.1 apparent thermal conductivity, $\lambda$-the ratio of the specimen thickness to thermal resistance of the specimen. It is calculated as follows:

$$
\begin{equation*}
\lambda=L / R(W / \mathrm{m} \cdot \mathrm{k}) \text { or }\left(\mathrm{Btu} \cdot{\left.\mathrm{in} . / \mathrm{ft}^{2} \cdot \mathrm{~h} \cdot \mathrm{~F}\right)}^{2}\right. \tag{1}
\end{equation*}
$$

3.2.1.1 Discussion-For this type of material an expression for the apparent thermal conductivity as a function of density is:

$$
\begin{equation*}
\lambda=a+b D+c / D \tag{2}
\end{equation*}
$$

where $a, b, c=$ parameters characteristic of a product, and related to the conductivity of the gas, the conductivity of the solid and the conductivity due to radiation.(1) ${ }^{3}$

### 3.3 Symbols:

```
\(R \quad=\) thermal resistance, \(\left(\mathrm{m}^{2} \mathrm{~K} / \mathrm{W}\right)\) or \(\left(\mathrm{h} \cdot \mathrm{ft}{ }^{2} \mathrm{~F} / \mathrm{Btu}\right)\)
\(\lambda \quad=\) apparent thermal conductivity, \((\mathrm{W} / \mathrm{m} \cdot \mathrm{K})\) or (Btu•in/
        \(h \cdot \mathrm{ft}^{2} \mathrm{~F}\) )
\(Q / A=\) heat flow per unit area, \(\left(\mathrm{W} / \mathrm{m}^{2}\right)\) or \(\left(\mathrm{Btu} / \mathrm{h} \cdot \mathrm{ft}^{2}\right)\)
\(D \quad=\) bulk density of a specimen, \(\left(\mathrm{kg} / \mathrm{m}^{3}\right)\) or \(\left(\mathrm{lb} / \mathrm{ft}^{3}\right)\)
\(L=\) measured specimen thickness, (m) or (in.)
\(T=\) apparatus plate temperature, \((\mathrm{K})\) or \((\mathrm{F})\)
\(L^{\prime}=\) specimen thickness if the sample from which the
    specimen is selected does not recover to label
    thickness, (m) or (in.)
\(s \quad=\) estimate of the standard deviation for a set of data
    points
\(\Delta \quad=\) apparatus systematic error
\(\Psi \quad=\) overall uncertainty in a measured \(R\)-value
    3.3.1 Subscripts:
```

[^1]```
= signifies average of a lot
H = refers to hot surface
C = refers to cold surface
T = refers to test specimen
N = refers to nominal property for the product, as shown
    on the product label
    = refers to a set of data points
s = refers to a particular specimen
```


## 4. Significance and Use

4.1 This guide provides a method to determine the thermal performance of low-density blanket-type insulation. It may be used for the purposes of quality assurance, certification, or research.
4.2 The thermal resistance of low-density insulation depends significantly on the density, the thickness, and thermal conductivity. Typical low-density, mineral-fiber insulation for buildings may vary in density from one specimen to the next.
4.3 Thermal tests are time-consuming in comparison with density and thickness measurements. Low-density insulation material is produced in large quantities. A typical lot would be a truckload or the amount necessary to insulate a house.
4.4 The relatively low unit cost of this product and the relatively high cost of thermal resistance testing makes it cost-effective to test only a small percentage of the product area. It is recommended that there be a determination of the density that is representative of a lot by the measurement of the average density of a statistically representative sampling.
4.5 A fewer number of thermal measurements are then made to determine the apparent thermal conductivity at the previously determined representative density. The essential significance of this guide is that a large lot of variable material is best characterized by: (a) determining the representative density, and by $(b)$ determining the thermal property at this representative density with a small number of thermal measurements.
4.6 Building insulation products are commonly manufactured in thicknesses ranging from 19 to 330 mm ( 0.75 to 13 in .) inclusive. Experimental work has verified that there is a dependence of $\lambda_{\text {app }}$ on thickness for some low density materials.
4.7 The upper limit of test thickness for specimens evaluated using Test Methods C 177, C 518, and C 1114 is established based upon the apparatus design, overall dimensions, expected thermal resistivity level and desired target accuracy. The testing organization is responsible for applying these restrictions when evaluating a product to ensure that the results meet applicable product labels and any existing regulatory requirements.(2)
4.8 Extrapolation of the apparent thermal conductivity or the thermal resistance beyond the ranges of thickness or density of products tested is not valid.

## 5. Sampling

5.1 For low-density mineral-fiber insulation, a lot sample size of 75 to $150 \mathrm{ft}^{2}$ is recommended to determine the average density, $D_{\text {av }}$. Density is determined by using Test Method C 167; take care to avoid the use of damaged material.
5.2 In order to account for the variation in $\lambda$-value due to product density variability, measure a minimum of three " $\lambda$
versus $D$ " data points on three different samples. This represents nine data points for the " $\lambda$ versus $D$ " curve. Again, this " $\lambda$ versus $D$ " curve is developed to determine the $\lambda$-value at a particular representative density characteristic of a lot of material.
5.3 The size of a lot of material to be characterized, the amount of material measured for the representative values of density and thickness, and the frequency of tests all depend on the user's needs, which could be related to quality assurance by a manufacturer, certification, or research.

## 6. Procedure

6.1 This procedure uses nine $\left\{\lambda_{i} ; D_{i}\right\}$ data points all measured at the same hot and cold plate temperatures, to establish an interpolation equation for the determination of the $\lambda$-value at the average density, $D_{\text {av. }}$. That is, the subscript $i$ refers to the $i^{\text {th }}$ test point. The $D_{i}$ is the average density of the specimen within the apparatus meter-area. The thermal resistance at $L_{\mathrm{av}}$ and $D_{\mathrm{av}}$ is as follows:

$$
\begin{equation*}
R_{\mathrm{av}}=L_{\mathrm{av}} / \lambda_{\mathrm{av}} \tag{3}
\end{equation*}
$$

6.2 Before the set of "apparent thermal conductivity versus test density ( $\lambda_{i}$ versus $D_{i}$ )" data points can be measured on an apparatus, it is necessary to choose the test densities and thicknesses. Three procedures for this choice are described in Annex A1.
6.2.1 Procedure A—A single test specimen is compressed to obtain different densities (A1.2). This procedure offers the advantage of less test time to obtain three test points.
6.2.2 Procedure $B$-A different specimen is used for each test point (A1.3). This method has the advantage of a better statistical sampling with regard to material variability.
6.2.3 Procedure $C$-Test at $D_{\mathrm{av}}$ thereby eliminating the need for an interpolation (A1.4).
6.3 Obtain a test value for $\lambda$ at each of the three densities. These three sets of test values result in three equations of the form of Eq 2 in 3.2.2. These are solved simultaneously to determine the values of $a_{s}, b_{s}$, and $c_{s}$ corresponding to specimen $s$ (see A2.1.2).

Note 1—Small errors in the measured values of $\lambda$ will result in large variations in the values of $a, b$, and $c$. Even so, the uncertainty of the interpolated value of $\lambda$ will be comparable to the measured error in $\lambda$.
6.4 Whenever possible, calculate running averages for the specific product lot based on a number $N$ equal to 20 or more sets of product curve parameters $\left(a_{s} ; \mathrm{b}_{s} ; \mathrm{c}_{s}\right)$. Remember from 6.3 that each of these sets requires three test points (see A2.1.3).
6.4.1 A larger number $N$ results in more consistent values for $a, b$, and $c$; a smaller $N$ represents a more current data base.
6.5 In 6.3 a set of parameter values was calculated, and in 6.4 a running average was calculated. This section describes how to obtain an interpolation curve (or equivalently a set of interpolation curve parameters) for the next sample, $s$, when it has been possible to previously obtain a running average set, ( $\bar{a} ; \bar{b} ; \bar{c})$. The given values are the set $\{\bar{a} ; \bar{b} ; \bar{c}\}$ and the measured values of $\lambda_{i}$ at three densities, $D_{i}$.

Note 2—Parameter $c$ is expected to account for most of the variation in
the " $\lambda$ versus $D$ " curve from specimen to specimen. When the density is less than $16 \mathrm{~kg} / \mathrm{m}^{3}\left(1 \mathrm{lb} / \mathrm{ft}^{3}\right), c$ is the dominant parameter causing the variance of $\lambda$ from specimen to specimen. Then the previously determined values, $\bar{a}$, and $b$ are used, along with a measurement of $\lambda$ at a particular density, to calculate a value of $c$ for a particular specimen, $s$. In order to have a better estimate of the mean, the value of $c$ is thusly determined for three values of density resulting in the value $\bar{c}_{s}$. The interpolation to the $\lambda$ value at the average density, $D_{\mathrm{av}}$, is calculated as follows, using Eq 3 .

$$
\begin{equation*}
\lambda_{s}=\bar{a}+\bar{b} D_{a v}+\bar{c}_{s} / D_{a v} \tag{4}
\end{equation*}
$$

An example of this calculation is in A2.1.4
6.6 Compute the average value of $\lambda_{\mathrm{av}}$ based on as many values of $\lambda_{s}$ that have been determined. Remember from 6.3 and 6.5 that three test points are required to obtain a value for $\lambda_{\mathrm{av}}$. Common practice is to base an average $\lambda_{\mathrm{av}}$ on three values of $\lambda_{s}$.
6.7 Calculate the $R$-value, $R_{\mathrm{av}}$, of the product at the average density and thickness (see Section 5 and A1.1) as follows:

$$
\begin{equation*}
R_{\mathrm{av}}=L_{T} / \lambda_{\mathrm{av}} \tag{5}
\end{equation*}
$$

## 7. Report

7.1 The report shall contain the following information:
7.1.1 The values of the average thermal resistance, density and thickness, the sample size, and the supporting data.
7.1.2 The test methods used and the information on the values and uncertainties of apparent thermal conductivity and density that is required in Test Method C 167, C 177, C 518, or C 1114.
7.1.3 The procedure used to obtain the $\lambda$ versus $D$ curve along with the equation for the curve itself.

## 8. Precision and Bias

8.1 There are a number of ways to combine the systematic and random uncertainties that contribute to an overall uncertainty of a measured quantity. The following procedure is intended as a guideline.
8.2 The term precision is used in this guide in the sense of repeatability. The estimation of the standard deviation, $s$, for a set of measurements with a normal distribution is the plus and minus range about an average value or curve, within which $68 \%$ of the observations lie. The $s$ is used to quantify the precision.
8.3 The term bias as used in this guide represents the total uncertainty in a set of measurements, including apparatus systematic error, apparatus precision, and the material variability.
8.4 The apparatus precision is the variation that occurs when repeated observations are made on a single specimen or identical specimens. It is quantified by $s_{a}$, and it is required as input data from either Test Method C 177, C 518, or C 1114.(3)
8.5 The material variability is partly taken into account by the $\lambda$ versus $D$ curve. When different specimens are tested there will be an amount of variation about the average $\lambda$ versus $D$
curve in addition to the apparatus precision. This additional variation is here called the material variability and is designated by $s_{m}$.
8.6 The total "repeatability" uncertainty on a $\lambda$ versus $D$ graph will be the sum of the aforementioned uncertainties and is designated by $s_{\lambda}$.

$$
\begin{equation*}
s_{\lambda}=\left(s_{a}{ }^{2}+s_{m}{ }^{2}\right)^{0.5} \tag{6}
\end{equation*}
$$

8.7 In order to know what $s_{\lambda}$ is, it is necessary to plot a number of $\lambda$ versus $D$ test points. Twenty or more points are recommended. It is then possible to determine by a graphical or a mathematical method (see Annex A3) what is the $1 s$ band within which $68 \%$ of the points lie or what is the $2 s$ band within which $95 \%$ of the points lie.
8.8 When more than one apparatus is used to develop the $\lambda$ versus $D$ curve, there will be a difference between the average values on the same set of specimens due to a systematic difference among the apparatus.
8.9 The measured data from an apparatus have associated with it an estimate of the possible systematic error in $\lambda$ of that apparatus. It is designated by $\Delta_{\lambda}$ and is provided as input from Test Method C 177, C 518, or C 1114.
8.10 For the purposes of this guide the overall accuracy, $\Psi_{\lambda}$, of the reported $\lambda$-value is the sum of the overall repeatability ( $1 s$ for a $68 \%$ confidence band) and the apparatus systematic error.

$$
\begin{equation*}
\Psi_{\lambda}=s_{\lambda}+\Delta_{\lambda} \tag{7}
\end{equation*}
$$

8.11 The percent "precision and bias" uncertainties in the reported $R$-value is calculated as follows, based on Eq 1:

$$
\begin{equation*}
R_{\mathrm{av}}=L_{T} / \lambda_{\mathrm{av}} \tag{8}
\end{equation*}
$$

8.11.1 The estimate of the residual standard deviation of $L_{\mathrm{av}}$ and $\lambda_{\mathrm{av}}$ is made by statistical methods (see Annex A3). The percent residual standard deviation in the reported $R$-value is then:

$$
\begin{equation*}
\frac{s_{R}}{R_{\mathrm{av}}}=\left(\frac{s_{L}^{2}}{{L_{T}}^{2}}+\frac{s_{\lambda}^{2}}{\lambda_{r}^{2}}\right)^{0.5} \tag{9}
\end{equation*}
$$

8.11.2 In order to calculate the percent bias uncertainty in $R_{v}$, it is necessary to obtain from Test Method C 167 the estimate of systematic uncertainty in the measurement of $L_{\mathrm{av}}$. This is of the order of the resolution of the measurement device, and it is designated here by $\Delta_{L}$. For the purpose of this guide, the overall percent bias in the reported $R$-value is calculated as follows:

$$
\begin{equation*}
\frac{\Psi_{R}}{R_{\mathrm{av}}}=\left(\frac{\left(s_{L}+\Delta_{L}\right)^{2}}{L_{\mathrm{av}}{ }^{2}}+\frac{\left(s_{\lambda}+\Delta_{\lambda}\right)^{2}}{\lambda_{\mathrm{av}}{ }^{2}}\right)^{0.5} \tag{10}
\end{equation*}
$$

## 9. Keywords

9.1 blanket; low-density; mineral fiber; thermal resistance

## ANNEXES

## (Mandatory Information)

## A1. PROCEDURES TO DETERMINE THE TEST VALUES FOR DENSITY AND THICKNESS

## A1.1 General Considerations

A1.1.1 With reference to 5.2, the sample (lot) average mass per unit area, $W_{\mathrm{av}} / A$, and average recovered thickness, $L_{\mathrm{av}}$, are determined using Test Method C 167.

Case 1-If $L_{\mathrm{av}} \geq L_{N}$, the product-label nominal thickness, then the average density of $D_{\mathrm{av}}$ is defined by $D_{\mathrm{av}}=M_{\mathrm{av}} / A \cdot L_{N}$. In this case the basic test thickness, $L_{T}$ is $L_{N}$.

Case 2-If $L_{\mathrm{av}}<L_{N}$, then $D_{\mathrm{av}}=M_{\mathrm{av}} / A \cdot L_{\mathrm{av}}$. Here, $L_{T}$ is $L_{\mathrm{av}}$.
A1.1.2 In choosing the nine specimens for the thermal tests, it is desirable to have the range of the individual densities as small as possible. This ensures an accurate interpolation for the determination of $\lambda_{\mathrm{av}}$ (see 6.6).
A1.1.3 All densities and heat flux measurements shall be based on the metering area.

## A1.2 Procedure to Obtain a $\boldsymbol{\lambda}$ versus $\boldsymbol{D}$ Curve with Compression

A1.2.1 The objective in the choice of specimen thickness and density is to bracket $D_{\mathrm{av}}$.

Case 1-The recovered specimen thickness is greater than or equal to 1.1 times the test thickness, $L_{T}$ (see A1.1). Here, the same specimen is tested at $1.1 L_{T}, L_{T}$ and $0.9 L_{T}$.

Case 2-The recovered specimen thickness, $L_{T}^{\prime}$, is less than 1.1 $L_{T}$. Here, the specimen is tested at $L_{T}^{\prime}, 0.9 L_{T}^{\prime}$ and $0.8 L_{T}^{\prime}$.

Note A1.1- $D_{\text {av }}$ should be close to the center of the interpolation region in Case 1 and close to the edge in Case 2.

Note A1.2-Three data points are taken on three different specimens, making a total of nine data points (see Annex A2).

## A1.3 Guideline Procedure to Obtain an $\boldsymbol{\lambda}$ versus $\boldsymbol{D}$ Curve Without Compression

A1.3.1 Here, nine different specimens are used to obtain test points at the three different appropriate densities. This is in contrast to the procedure in A1.2 in which a single specimen is compressed to obtain test points of $\lambda$ at three different densities. It is important to choose each set of three specimens with densities so that the first is near the lower extreme, the second is near the median, and the third is near the upper extreme of the density range encountered in the determination of the average density ( 3.3 and 5.1). Once the $\lambda$ versus $D$ test points are obtained, the calculation is the same as that in 6.3-6.6. Note that A1.3 must not be used in a case where there is an extrapolation in density.

## A1.4 Procedure to Determine the $\lambda$-Value at $D_{\text {av }}$ Without Interpolation

A1.4.1 This procedure can be used only when the recovered density of the specimen is less than or equal to the average density, $D_{\text {av }}$, and when the specimen is compressible. The basic idea is to have the specimen at a greater-than-test thickness so that when it is compressed to the test thickness the test density is $D_{\mathrm{av}}$.

A1.4.2 As an alternative to the procedures in A1.2 and A1.3, nine $\lambda$-values are measured on nine different specimens by the aforementioned procedure at densities as close as possible to $D_{\mathrm{av}}$. The $\lambda$-value at $D_{\mathrm{av}}$ is then the average of these, and no interpolation is necessary. (That is, the procedure outlined in 6.3-6.5 is not needed.)

## A2. SAMPLE CALCULATION

A2.1 This calculation follows the procedure in Section 6 and in A1.2.

A2.1.1 The input data for a specimen, $s$, are as follows:

$$
\begin{gathered}
L_{a v}>L_{N}, \text { so } L_{T}=L_{N} \\
L_{N}=88.9 \mathrm{~mm}(3.5 \mathrm{in} .) \\
D_{\mathrm{av}}=8.89 \mathrm{~kg} / \mathrm{m}^{3}\left(0.555 \mathrm{lb} / \mathrm{ft}^{3}\right)\left(\text { at } L_{T}=3.5 \mathrm{in} .\right) \\
\lambda-\text { units are } \mathrm{W} /\left(\mathrm{m} \cdot{ }^{\circ} \mathrm{C}\right) \text { or Btu} \cdot \mathrm{in} . /\left(\mathrm{h} \cdot \mathrm{ft}^{2} \cdot{ }^{\circ} \mathrm{F}\right) \\
\lambda_{1}=0.0454(0.315) \text { at } D_{1}=8.01 \mathrm{~kg} / \mathrm{m}^{3}=\left(0.500 \mathrm{lb} / \mathrm{ft}^{3}\right) ; L_{1} \approx 1.1 L_{T} \\
\lambda_{2}=0.0433(0.300) D_{2}=8.89 \mathrm{~kg} / \mathrm{m}^{3}\left(0.555 \mathrm{lb} / \mathrm{ft}^{3}\right) ; L_{2}=L_{T} \\
\lambda_{3}=0.0415(0.288) \text { at } D_{3}=10.01 \mathrm{~kg} / \mathrm{m}^{3}\left(0.625 \mathrm{lb} / \mathrm{ft}^{3}\right) ; L_{3} \approx 0.9 L_{T}
\end{gathered}
$$

A2.1.2 Using Eq 2 from 3.2.2, the three simultaneous equations to be solved in inch-pound units are as follows:

$$
\begin{aligned}
& 0.315=\mathrm{a}_{s}+\mathrm{b}_{s}(0.500)+\mathrm{c}_{s} /(0.500) \\
& 0.300=\mathrm{a}_{s}+\mathrm{b}_{s}(0.555)+\mathrm{c}_{s} /(0.555) \\
& 0.288=\mathrm{a}_{s}+\mathrm{b}_{s}(0.625)+\mathrm{c}_{s} /(0.625)
\end{aligned}
$$

Simultaneous solution yields the following results:

$$
\begin{gathered}
\lambda=(-0.083)+(0.234) D+(0.141) / D \\
a_{s}=-0.083 ; b_{s}=0.234 ; c_{s}=0.141
\end{gathered}
$$

A2.1.3 Similar values for the last 20 specimens are used to calculate the running averages- $\bar{a}, \bar{b}$ and $\bar{c}$, which define the standard product curve. Typical running average values are $\bar{a}$ $=0.170, \bar{b}=0.005$, and $\bar{c}=0.070$. Alternatively, a multiple linear regression could be used to determine these values.

A2.1.4 The previously determined values, $a$ and $b$, are used along with the data in A2.1.1 to calculate three values of $c_{s}$, which are designated $c_{s i}$ where $i$ refers to a data point at a particular density.

$$
\begin{gathered}
\lambda_{s i}=\bar{a}+\bar{b} D_{i}+c_{s i} / D_{i} \\
0.315=(0.17)+(0.005)(0.500)+\left(\mathrm{c}_{s 1} / 0.500\right) \\
c_{s 1}=0.0712 \\
0.300=(0.17)+(0.005)(0.555)+\left(c_{s 2} / 0.555\right) \\
c_{s 2}=0.0706
\end{gathered}
$$

$$
\begin{gathered}
0.288=(0.17)+(0.005)(0.625)+\left(c_{s 3} / 0.625\right) \\
c_{s 3}=0.0718 \\
\bar{c}_{s}=(1 / 3) c_{s 1}+c_{s 2}+c_{s 3}=0.0712
\end{gathered}
$$

The value of $\bar{c}_{s}$ is used along with the previously determined values of $\bar{a}$ and $\bar{b}$ to define the interpolation curve for the specimen. The value of $\lambda_{s}$ at $D_{\mathrm{av}}$ is calculated as follows:

$$
\begin{gathered}
D_{\mathrm{av}}=0.550 \\
\lambda_{s}=\bar{a}+\bar{b} D_{\mathrm{av}}+\bar{c}_{s} / D_{\mathrm{av}} \\
=(0.17)+(0.005)(0.550)+(0.0712) /(0.550)=0.3022
\end{gathered}
$$

A2.1.5 This procedure is repeated on the other two specimens. The average of the three specimen thicknesses and the average of the three calculated thermal conductivities are used to calculate the thermal resistance of the product. If this average value of $\lambda_{\mathrm{av}}$ is 0.314 , and the average thickness is 3.5 in., then $R_{\mathrm{av}}=(3.5 / 0.314)=11.15\left(\mathrm{~h} \cdot \mathrm{ft}^{2} \cdot{ }^{\circ} \mathrm{F} / \mathrm{Btu}\right)$.

A2.2 An alternative calculation procedure is to use least squares regression analysis. There are many computer and hand calculator programs that are readily available but each program may be slightly different so only general guidelines can be given here.

A2.2.1 Use the same expression for apparent thermal conductivity, which is:

$$
\lambda=a+b D+c / D \text { as described in 3.2.2 }
$$

A2.2.2 A minimum of nine data pairs must be used. The data can be obtained by either testing nine different specimens or by testing three different thicknesses.

A2.2.3 Once the parameters $a, b$ and $c$ are determined, an apparent thermal conductivity can be calculated for any desired density. While this method is very accurate for interpolated values, care must be taken to avoid extrapolation because this can cause significant errors.

## A3. COMMENTS ON PRECISION AND BIAS

A3.1 In 8.11.1 reference is made to the residual standard deviation, $s \lambda$ with respect to the $\lambda$ versus $D$ graph. It can be estimated graphically by drawing a confidence band within which $68 \%$ of the data points lie. One half of the width of this band is $s_{\lambda}$. Mathematically, $s_{\lambda}$ is calculated as follows:

A3.1.1 Let $f(D)=\bar{a}+\bar{b} D+\bar{c} / D$ represent the product curve based on all the points on the graph. This set $(a, b, c)$ can be based on a running average, or more exactly it can be based on a least squares fit. The residual for each data point is defined as $\Delta \lambda_{i}=\lambda_{i}-f\left(D_{i}\right)$

$$
s_{\lambda}=\left(\frac{\sum\left(\Delta \lambda_{\mathrm{i}}\right)^{2}}{N-3}\right)^{0.5}
$$

where $(N-3)$ is used since there are three parameters.(4)
A3.2 The form of the equation to describe the functional dependences of $\lambda$ on $D$ (see 4.2.3) could be different for different materials. A two-parameter form $(\lambda=a+b / D)$ has been found to work well in terms of the bias of an interpolation over the limited density range from 8 to $16 \mathrm{~kg} / \mathrm{m}^{3}(0.5$ to 1.0 $\mathrm{lb} / \mathrm{ft}^{3}$ ). (5),(6) An additional parameter could be included to account for the dependence of $\lambda$ on thickness as well as on density. The additional precision afforded by adding this additional parameter would be negligible compared with the apparatus and material precision. Likewise, the contribution of rounding errors in the calculation are comparatively negligible.

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[^0]:    ${ }^{1}$ This guide is under the jurisdiction of ASTM Committee C16 on Thermal Insulation and is the direct responsibility of Subcommittee C16.30 on Thermal Measurement.

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[^1]:    ${ }^{3}$ The boldface numbers in parentheses refer to a list of references at the end of this standard.

