



Standard Test Method for Young's Modulus, Shear Modulus, and Poisson's Ratio For Ceramic Whitewares by Resonance¹

This standard is issued under the fixed designation C 848; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ϵ) indicates an editorial change since the last revision or reapproval.

1. Scope

1.1 This test method covers the determination of the elastic properties of ceramic whiteware materials. Specimens of these materials possess specific mechanical resonance frequencies which are defined by the elastic moduli, density, and geometry of the test specimen. Therefore the elastic properties of a material can be computed if the geometry, density, and mechanical resonance frequencies of a suitable test specimen of that material can be measured. Young's modulus is determined using the resonance frequency in the flexural mode of vibration. The shear modulus, or modulus of rigidity, is found using torsional resonance vibrations. Young's modulus and shear modulus are used to compute Poisson's ratio, the factor of lateral contraction.

1.2 All ceramic whiteware materials that are elastic, homogeneous, and isotropic may be tested by this test method.² This test method is not satisfactory for specimens that have cracks or voids that represent inhomogeneities in the material; neither is it satisfactory when these materials cannot be prepared in a suitable geometry.

NOTE 1—Elastic here means that an application of stress within the elastic limit of that material making up the body being stressed will cause an instantaneous and uniform deformation, which will cease upon removal of the stress, with the body returning instantly to its original size and shape without an energy loss. Many ceramic whiteware materials conform to this definition well enough that this test is meaningful.

NOTE 2—Isotropic means that the elastic properties are the same in all directions in the material.

¹ This test method is under the jurisdiction of ASTM Committee C21 on Ceramic Whitewares and Related Products and is the direct responsibility of Subcommittee C21.03 on Methods for Whitewares and Environmental Concerns.

Current edition approved Feb. 15, 2006. Published February 2006. Originally approved in 1976. Last previous edition approved in 1999 as C 848 – 88 (1999).

² Spinner, S., and Tefft, W. E., "A Method for Determining Mechanical Resonance Frequencies and for Calculating Elastic Moduli from These Frequencies," *Proceedings*, ASTM, 1961, pp. 1221–1238.

1.3 A cryogenic cabinet and high-temperature furnace are described for measuring the elastic moduli as a function of temperature from -195 to 1200°C .

1.4 Modification of the test for use in quality control is possible. A range of acceptable resonance frequencies is determined for a piece with a particular geometry and density. Any specimen with a frequency response falling outside this frequency range is rejected. The actual modulus of each piece need not be determined as long as the limits of the selected frequency range are known to include the resonance frequency that the piece must possess if its geometry and density are within specified tolerances.

1.5 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory limitations prior to use.*

2. Summary of Test Method

2.1 This test method measures the resonance frequencies of test bars of suitable geometry by exciting them at continuously variable frequencies. Mechanical excitation of the specimen is provided through use of a transducer that transforms an initial electrical signal into a mechanical vibration. Another transducer senses the resulting mechanical vibrations of the specimen and transforms them into an electrical signal that can be displayed on the screen of an oscilloscope to detect resonance. The resonance frequencies, the dimensions, and the mass of the specimen are used to calculate Young's modulus and the shear modulus.

3. Significance and Use

3.1 This test system has advantages in certain respects over the use of static loading systems in the measurement of ceramic whitewares.

3.1.1 Only minute stresses are applied to the specimen, thus minimizing the possibility of fracture.

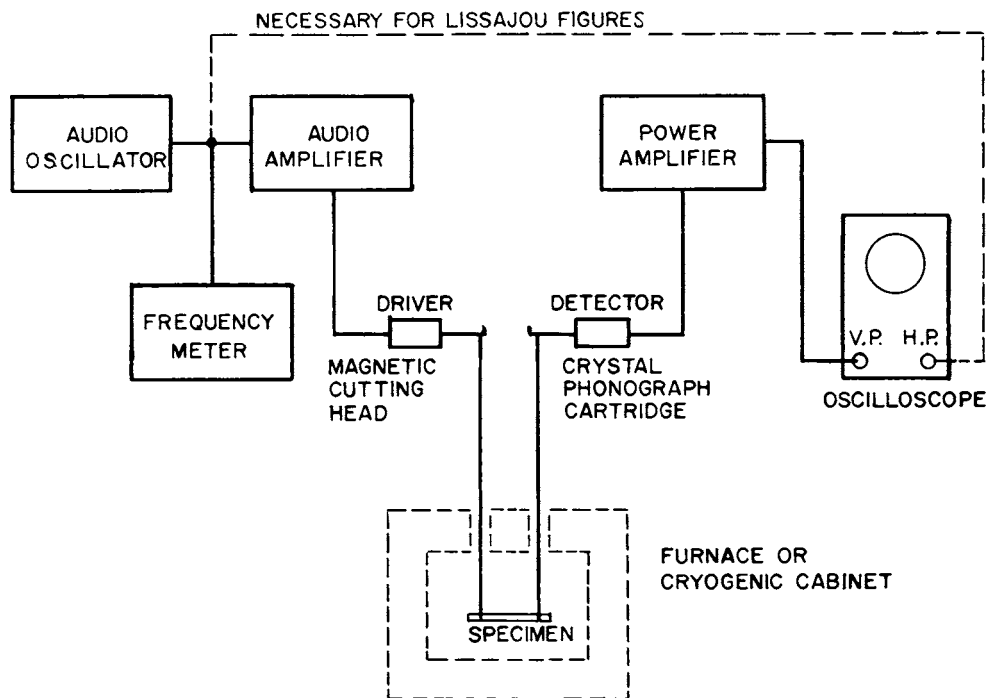


FIG. 1 Block Diagram of Apparatus

3.1.2 The period of time during which stress is applied and removed is of the order of hundreds of microseconds, making it feasible to perform measurements at temperatures where delayed elastic and creep effects proceed on a much-shortened time scale.

3.2 This test method is suitable for detecting whether a material meets specifications, if cognizance is given to one important fact: ceramic whiteware materials are sensitive to thermal history. Therefore, the thermal history of a test specimen must be known before the moduli can be considered in terms of specified values. Material specifications should include a specific thermal treatment for all test specimens.

4. Apparatus

4.1 The test apparatus is shown in Fig. 1. It consists of a variable-frequency audio oscillator, used to generate a sinusoidal voltage, and a power amplifier and suitable transducer to convert the electrical signal to a mechanical driving vibration. A frequency meter monitors the audio oscillator output to provide an accurate frequency determination. A suitable suspension-coupling system cradles the test specimen, and another transducer acts to detect mechanical resonance in the specimen and to convert it into an electrical signal which is passed through an amplifier and displayed on the vertical plates of an oscilloscope. If a Lissajous figure is desired, the output of the oscillator is also coupled to the horizontal plates of the oscilloscope. If temperature-dependent data are desired, a suitable furnace or cryogenic chamber is used. Details of the equipment are as follows:

4.2 *Audio Oscillator*, having a continuously variable frequency output from about 100 to at least 20 kHz. Frequency drift shall not exceed 1 Hz/min for any given setting.

4.3 *Audio Amplifier*, having a power output sufficient to ensure that the type of transducer used can excite any specimen the mass of which falls within a specified range.

4.4 *Transducers*—Two are required; one used as a driver may be a speaker of the tweeter type or a magnetic cutting head or other similar device, depending on the type of coupling chosen for use between the transducer and the specimen. The other transducer, used as a detector, may be a crystal or magnetic reluctance type of phonograph cartridge. A capacitive pickup may be used if desired. The frequency response of the transducer shall be as good as possible with at least a 6.5-kHz bandwidth before 3-dB power loss occurs.

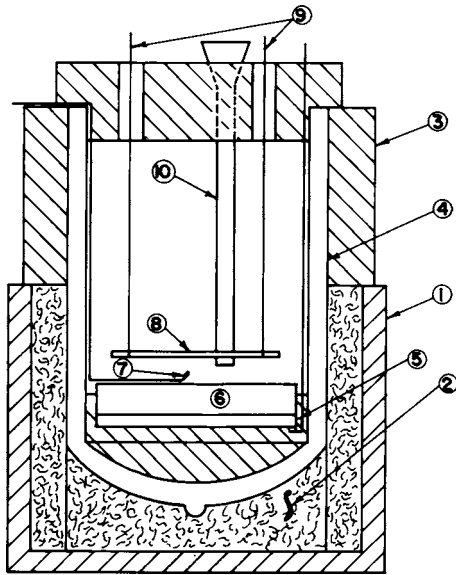
4.5 *Power Amplifier*, in the detector circuit shall be impedance matched with the type of detector transducer selected and shall serve as a prescope amplifier.

4.6 *Cathode-Ray Oscilloscope*, shall be any model suitable for general laboratory work.

4.7 *Frequency Counter*, shall be able to measure frequencies to within ± 1 Hz.

4.8 If data at elevated temperatures are desired, a furnace shall be used that is capable of controlled heating and cooling. It shall have a specimen zone 180 mm in length, which will be uniform in temperature within $\pm 5^\circ\text{C}$ throughout the range of temperatures encountered in testing.

4.9 For data at cryogenic temperatures, any chamber shall suffice that is capable of controlled heating, frost-free, and



- 1—Cylindrical glass jar
- 2—Glass wool
- 3—Plastic foam
- 4—Vacuum jar
- 5—Heater disk
- 6—Copper plate
- 7—Thermocouple
- 8—Sample
- 9—Suspension wires
- 10—Fill port for liquid

FIG. 2 Detail Drawing of Suitable Cryogenic Chamber

uniform in temperature within $\pm 5^{\circ}\text{C}$ over the length of the specimen at any selected temperature. A suitable cryogenic chamber³ is shown in Fig. 2.

4.10 Any method of specimen suspension shall be used that is adequate for the temperatures encountered in testing and that shall allow the specimen to vibrate without significant restriction. Common cotton thread, silica glass fiber thread, Nichrome, or platinum wire may be used. If metal wire suspension is used in the furnace, coupling characteristics will be improved if, outside the temperature zone, the wire is coupled to cotton thread and the thread is coupled to the transducer. If specimen supports of other than the suspension type are used, they shall meet the same general specifications.

5. Test Specimens

5.1 Prepare the specimens so that they are either rectangular or circular in cross section. Either geometry can be used to measure both Young's modulus and shear modulus. However, great experimental difficulties in obtaining torsional resonance frequencies for a cylindrical specimen usually preclude its use in determining shear modulus, although the equations for computing shear modulus with a cylindrical specimen are both simpler and more accurate than those used with a prismatic bar.

³ Smith, R. E., and Hagy, H. E., "A Low Temperature Sonic Resonance Apparatus for Determining Elastic Properties of Solids," Internal Report 2195, Corning Glass Works, April 1961.

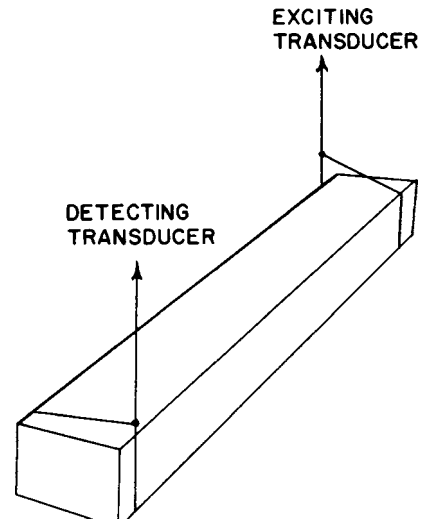


FIG. 3 Specimen Positioned for Measurement of Flexural and Torsional Resonance Frequencies Using Thread or Wire Suspension

5.2 Resonance frequencies for a given specimen are functions of the bar dimensions as well as its density and modulus; therefore, dimensions should be selected with this relationship in mind. Make selection of size so that, for an estimated modulus, the resonance frequencies measured will fall within the range of frequency response of the transducers used. Representative values of Young's modulus are 10×10^6 psi (69 GPa) for vitreous triaxial porcelains and 32×10^6 psi (220 GPa) for 85 % alumina porcelains. Recommended specimen sizes are 125 by 15 by 6 mm for bars of rectangular cross section and 125 by 10 to 12 mm for those of circular cross section. These specimen sizes should produce a fundamental flexural resonance frequency in the range from 1000 to 2000 Hz. Specimens shall have a minimum mass of 5 g to avoid coupling effects: any size of specimen that has a suitable length-to-cross section ratio in terms of frequency response and meets the mass minimum may be used. Maximum specimen size and mass are determined primarily by the test system's energy and space capabilities.

5.3 Finish specimens using a fine grind, 400 grit or smaller. All surfaces shall be flat and opposite surfaces shall be parallel within 0.02 mm.

6. Procedure

6.1 *Procedure A, Room Temperature Testing*—Position the specimen properly (see Figs. 3 and 4). Activate the equipment so that power adequate to excite the specimen is delivered to the driving transducer. Set the gain of the detector circuit high enough to detect vibration in the specimen and to display it on the oscilloscope screen with sufficient amplitude to measure accurately the frequency at which the signal amplitude is maximized. Adjust the oscilloscope so that a sharply defined horizontal baseline exists when the specimen is not excited. Scan frequencies with the audio oscillator until specimen resonance is indicated by a sinusoidal pattern of maximum amplitude on the oscilloscope. Find the fundamental mode of vibration in flexure, then find the first overtone in flexure (Note 3). Establish definitely the fundamental flexural mode by

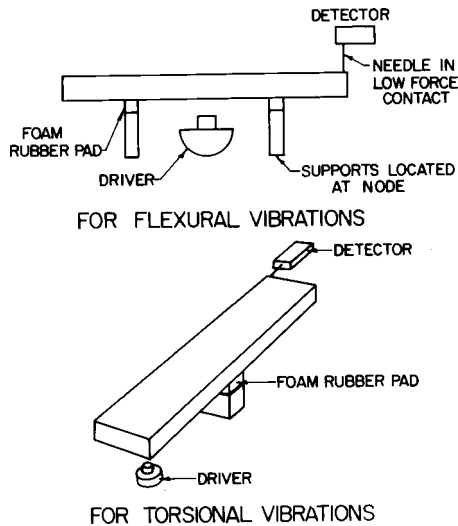
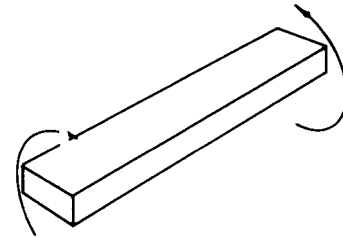


FIG. 4 Specimen Positioned for Measurement of Flexural and Torsional Resonance Frequencies Using “Tweeter” Exciter



FUNDAMENTAL IN FLEXURE



FUNDAMENTAL IN TORSION

FIG. 5 Some Modes of Mechanical Vibration in a Bar

positioning the detector at the appropriate nodal position of the specimen (see Fig. 5). At this point, the amplitude of the resonance signal will decrease to zero. The ratio of the first overtone frequency to the fundamental frequency will be approximately 2.70 to 2.75. If a determination of the shear modulus is to be made, offset the coupling to the transducers so that the torsional mode of vibration may be detected (see Fig. 3). Find the fundamental resonance vibration in this mode. Identify the torsional mode by centering the detector with respect to the width of the specimen and observing that the amplitude of the resonance signal decreases to zero; if it does not, the signal is an overtone of flexure or a spurious frequency generated elsewhere in the system. Dimensions and weight of the specimen may be measured before or after the test. Measure the dimensions with a micrometer caliper capable of an accuracy of ± 0.01 mm; measure the weight with a balance capable of ± 10 -g accuracy.

NOTE 3—It is recommended that the first overtone in flexure be determined for both rectangular and cylindrical specimens. This is useful in establishing the proper identification of the fundamental, particularly when spurious frequencies inherent in the system interfere (as, for example, when greater excitation power and detection sensitivity are required for work with a specimen that has a poor response). The fundamental and overtone are properly identified by showing them to be in the correct numerical ratio, and by demonstrating the proper locations of the nodes for each. Spinner and Tefft recommend using only the fundamental in flexure when computing Young’s modulus for a rectangular bar because of the approximate nature of Pickett’s theory. However, for the nominal size of bar specified, the values of Young’s modulus computed using Eq 1 and Eq 2 will agree within 1 %. When the correction factor, T_2 , is greater than 2 %, Eq 2 should not be used.

6.2 Procedure B, Elevated Temperature Testing—Determine the mass, dimensions, and frequencies at room temperature in air as outlined in 6.1. Place the specimen in the furnace and adjust the driver-detector system so that all the frequencies to be measured can be detected without further adjustment. Determine the resonant frequencies at room temperature in the furnace cavity with the furnace doors closed, and so forth, as will be the case at elevated temperatures. Heat the furnace at a

controlled rate that does not exceed $150^\circ\text{C}/\text{h}$. Take data at 25° intervals or at 15-min intervals as dictated by heating rate and specimen composition. Follow the change in resonance frequencies with time closely to avoid losing the identity of each frequency. (The overtone in flexure and the fundamental in torsion may be difficult to differentiate if not followed closely; spurious frequencies inherent in the system may also appear at temperatures above 600°C using certain types of suspensions, particularly wire.) If desired, data may also be taken on cooling; it must be remembered, however, that high temperatures may damage the specimen, by serious warping for example, making subsequent determinations of doubtful value.

6.3 Procedure C—Cryogenic Temperature Testing—Determine the weight, dimensions, and resonance frequencies in air at room temperature. Measure the resonance frequencies at room temperature in the cryogenic chamber. Take the chamber to the minimum temperature desired (Note 4), monitoring frequencies as the chamber is cooled. Allow the specimen to stabilize at minimum temperature for at least 15 min. Take data on heating. Heating rate should not exceed $50^\circ\text{C}/\text{h}$ and data may be taken at intervals of 10 min or 15°C or as desired.

NOTE 4—Precautions should be taken to remove water vapor from the chamber by flushing with dry nitrogen gas before chilling so that frost deposits on the specimen do not cause anomalous results.

7. Calculation

7.1 Young’s Modulus:

7.1.1 For the fundamental in flexure of a rectangular bar (Note 5):

$$E = 96.517 (L^3 / \text{lb}^3) T_1 w f^2 \times 10^{-8} \tag{1}$$

where:

- E = Young’s modulus, kgf/cm^2 ;
- L = length of the bar, cm;

b = width of the bar, cm;
 t = thickness of the bar, cm;
 w = weight of the bar, g;
 f = resonance frequency of bar, Hz; and
 T_1 = correction factor for fundamental flexural mode to account for finite thickness of bar, Poisson's ratio, and so forth. (See Table 1 for a plot of T_1 as a function of bar dimensions and Poisson's ratio.)

NOTE 5—Pickett⁴ and Goens⁵ have analyzed the relationship of the general equation for modulus of elasticity:

$$M = K wf_M^2$$

where:

M = elastic modulus,
 K = factor whose value depends on the dimensions of the bar and the particular characteristic vibration being investigated,
 w = weight of the bar, and
 f_M = frequency, Hz, of the characteristic vibration.

From this equation they have developed specific equations for use in the flexural and torsional modes of vibration (Eq 1-6).

7.1.2 For the first overtone in flexure of a rectangular bar (Note 5):

$$E = 12.703 (L^3 / bt_3) T_2 wf^2 \times 10^{-8} \quad (2)$$

where:

T_2 = correction factor for first overtone in flexure. (See Table 2 for a plot of T_2 as a function of bar dimensions and Poisson's ratio.)

7.1.3 For the fundamental in flexure of a rod of circular cross section (Note 5):

$$E = 1.6408 (L^3 / D^4) T_1 wf^2 \times 10^{-6} \quad (3)$$

where:

D = diameter of rod, cm.

7.1.4 For the first overtone in flexure of a rod of circular cross section (Note 5):

$$E = 21.567 (L^3 / D^4) T_2 wf^2 \times 10^{-8} \quad (4)$$

7.2 Shear Modulus:

7.2.1 For the fundamental torsion of a rectangular bar (Note 5):⁶

$$G = (10.197 B wf^2 \times 10^{-7}) / (1 + A) \quad (5)$$

⁴ Pickett, G., "Equations for Computing Elastic Constants from Flexural and Torsional Resonant Frequencies of Vibration of Prisms and Cylinders," *Proceedings*, ASTM, Vol. 45, 1945, pp. 846-865.

⁵ Goens, E., "Über die Bestimmung des Elastizitätsmoduls von Stüben mit Hilfe von Biegungsschwingungen," *Annalen der Physik, B. Folge*, Band 11, 1931, pp. 649-678.

⁶ Shear modulus correction taken from: Spinner, S., and Valore, R. C., "Comparison of Theoretical and Empirical Relations Between the Shear Modulus and Torsional Resonance Frequencies for Bars and Rectangular Cross Sections," *Journal of Research*, Nat. Bureau Standards, Vol 60, 1958, RP2861, p. 459.

TABLE 1 Correction Factor, T_1 for the Fundamental Mode of Flexural Vibration

$K(t/L)^A$	Poisson's Ratio			
	0.15	0.20	0.25	0.30
0.000	1.000 000	1.000 000	1.000 000	1.000 000
0.005	1.002 029	1.002 053	1.002 077	1.002 100
0.010	1.008 102	1.008 199	1.008 295	1.008 388
0.015	1.018 186	1.018 405	1.018 619	1.018 826
0.020	1.032 233	1.032 618	1.032 994	1.033 360
0.025	1.050 174	1.050 765	1.051 344	1.051 916
0.030	1.071 920	1.072 753	1.073 577	1.074 393
0.035	1.097 378	1.098 495	1.099 599	1.100 694
0.040	1.126 452	1.127 884	1.129 302	1.130 711
0.045	1.159 039	1.160 817	1.162 582	1.164 337
0.050	1.195 038	1.197 191	1.199 332	1.201 464
0.055	1.234 351	1.236 906	1.239 449	1.241 986
0.060	1.276 886	1.279 865	1.282 836	1.285 807
0.065	1.322 555	1.325 980	1.329 403	1.332 832
0.070	1.371 276	1.375 167	1.379 065	1.382 977
0.075	1.422 974	1.427 352	1.431 747	1.436 169
0.080	1.477 584	1.482 465	1.487 380	1.492 337
0.085	1.535 043	1.540 446	1.545 901	1.551 420
0.090	1.595 298	1.601 240	1.607 259	1.613 367
0.095	1.658 300	1.664 800	1.671 405	1.678 129
0.100	1.724 007	1.731 082	1.738 298	1.745 669
0.105	1.792 382	1.800 052	1.807 904	1.815 954
0.110	1.863 393	1.871 677	1.880 193	1.888 956
0.115	1.937 012	1.945 932	1.955 140	1.964 653
0.120	2.013 216	2.022 795	2.032 727	2.043 030
0.125	2.091 985	2.102 247	2.112 937	2.124 075
0.130	2.173 303	2.184 276	2.195 762	2.207 781
0.135	2.257 157	2.268 871	2.281 194	2.294 146
0.140	2.343 539	2.356 026	2.369 231	2.383 174
0.145	2.432 439	2.445 736	2.459 873	2.474 869
0.150	2.523 855	2.538 002	2.553 126	2.569 244

^A For cylindrical rods, $K = 1/4$. For rectangular bars, $K = 1/3.4641$.

where:

G = shear modulus, kgf/cm²

$$B = \frac{4L}{bt} \left[\frac{b^2 t + t^2 b}{4(t/b) - 2.52(t/b)^2 + 0.21(t/b)^6} \right] \quad (6)$$

and

A = correction factor dependent on the width-to-thickness ratio of the test specimen. (See Fig. 6 for a plot of A as a function of the width-to-thickness ratio.)

7.2.2 For the fundamental torsion of a cylindrical rod:

$$G = (2 L wf^2 \times 10^{-3}) / \pi r^2 \quad (7)$$

where:

G = shear modulus, kgf/cm² and

r = radius of rod, cm.

7.3 Poisson's Ratio:

$$\mu = (E/2G) - 1 \quad (8)$$

where:

μ = Poisson's ratio,

E = Young's modulus, and

G = shear modulus.

7.4 Calculation of moduli at elevated and cryogenic temperatures:

TABLE 2 Correction Factor, T_2 , for the First Overtone of Flexural Vibration

$K(t/L)^A$	Poisson's Ratio			
	0.15	0.20	0.25	0.30
0.0000	1.000 000	1.000 000	1.000 000	1.000 000
0.0025	1.001 422	1.001 445	1.001 467	1.001 489
0.0050	1.005 683	1.005 774	1.005 863	1.005 952
0.0075	1.012 770	1.012 973	1.013 174	1.013 373
0.0100	1.022 660	1.023 019	1.023 376	1.023 728
0.0125	1.035 326	1.035 885	1.036 440	1.036 987
0.0150	1.050 734	1.051 537	1.052 327	1.053 108
0.0175	1.068 844	1.069 926	1.070 995	1.072 054
0.0200	1.089 613	1.091 015	1.092 401	1.093 775
0.0225	1.112 996	1.114 756	1.116 497	1.118 223
0.0250	1.138 949	1.141 103	1.143 235	1.145 352
0.0275	1.167 429	1.170 012	1.172 573	1.175 115
0.0300	1.198 394	1.201 441	1.204 465	1.207 470
0.0325	1.231 807	1.235 353	1.238 874	1.242 376
0.0350	1.267 633	1.271 711	1.275 764	1.279 800
0.0375	1.305 844	1.310 487	1.315 106	1.319 710
0.0400	1.346 414	1.351 656	1.356 876	1.362 084
0.0425	1.389 325	1.395 199	1.401 055	1.406 905
0.0450	1.434 563	1.441 104	1.447 633	1.454 162
0.0475	1.482 122	1.489 364	1.496 602	1.503 850
0.0500	1.532 000	1.539 980	1.547 966	1.555 973
0.0525	1.584 202	1.592 958	1.601 732	1.610 540
0.0550	1.638 741	1.648 312	1.657 917	1.667 572
0.0575	1.695 636	1.706 063	1.716 543	1.727 092
0.0600	1.754 912	1.766 240	1.777 642	1.789 136
0.0625	1.816 603	1.828 880	1.841 254	1.853 748
0.0650	1.880 751	1.894 027	1.907 428	1.920 979
0.0675	1.947 405	1.961 734	1.976 222	1.990 892
0.0700	2.016 625	2.032 066	2.047 703	2.063 560
0.0725	2.088 477	2.105 095	2.121 950	2.139 068
0.0750	2.163 041	2.180 906	2.199 055	2.217 513
0.0775	2.240 405	2.259 595	2.279 119	2.299 006
0.0800	2.320 672	2.341 270	2.362 262	2.383 673
0.0825	2.403 955	2.426 056	2.448 613	2.471 654
0.0850	2.490 383	2.514 090	2.538 324	2.563 111
0.0875	2.580 098	2.605 527	2.631 559	2.658 220
0.0900	2.673 262	2.700 540	2.728 506	2.757 184

^A For cylindrical rods, $K = 1/4$. For rectangular bars $K = 1/3.4641$.

$$M_T = M_o [f_T / f_o]^2 [1 / (1 + \alpha \Delta T)] \quad (9)$$

where:

M_T = modulus at temperature T ;

M_o = modulus at room temperature;

f_T = resonance frequency in furnace or cryogenic chamber at temperature T ;

f_o = resonance frequency at room temperature in furnace or cryogenic chamber;

α = average linear thermal expansion coefficient of specimen from room temperature to test temperature, cm/cm·°C; and

ΔT = temperature differential, test temperature from room temperature.

8. Report

8.1 Report the following information:

8.1.1 Identification of specific tests performed and apparatus used,

8.1.2 Complete description of material(s) tested stating its composition and any treatment to which it has been subjected. Comments on surface finish, edge conditions, and so forth shall be included where pertinent,

8.1.3 Name of person requesting test,

8.1.4 Laboratory notebook number and page on which test data is recorded and file number if used, and

8.1.5 Numerical values obtained for Young's modulus, shear modulus, and Poisson's ratio.

9. Precision and Bias

9.1 Precision and bias on the order of 1 % for moduli and 10 % for Poisson's ratio is possible if all tolerances on dimensions are observed, resonance frequencies are measured with a frequency counter, and the weight of the specimen is measured within 10 mg.

10. Keywords

10.1 ceramic whitewares; Poisson's ratio; resonance; shear modulus; Young's modulus

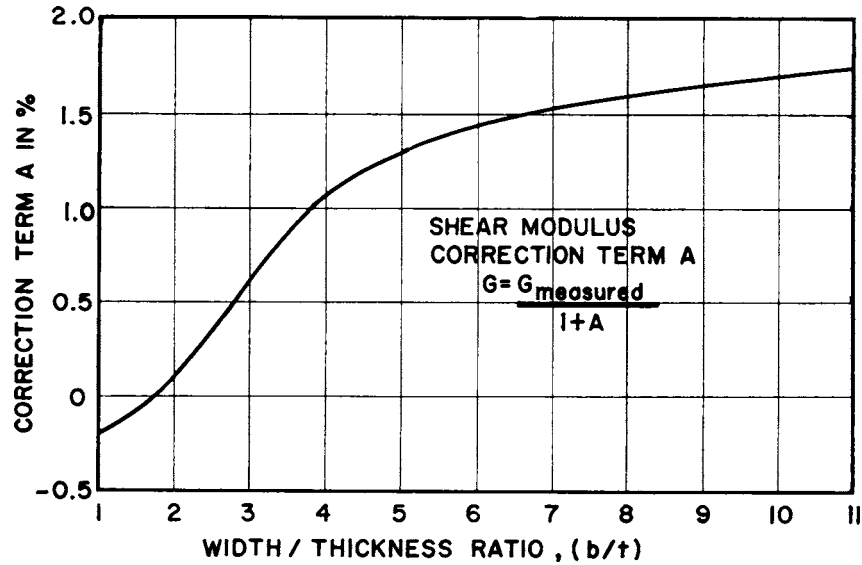


FIG. 6 Plot of the Shear Modulus Correction Term A

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