British Standard

BRITISH STANDARD

BS 8110-3:1985

Structural use of concrete —

Part 3: Design charts for singly reinforced beams, doubly reinforced beams and rectangular columns

UDC 624.012.3/.4 + 691.3

Committees responsible for this British Standard

The preparation of this British Standard was entrusted by the Civil Engineering and Building Structures Standards Committee (CSB/-) to Technical Committee CSB/39, upon which the following bodies were represented:

Association of Consulting Engineers
British Aggregate Construction Materials Industries
British Precast Concrete Federation Ltd.
British Railways Board
British Ready Mixed Concrete Association
British Reinforcement Manufacturers' Association
British Steel Industry
Building Employers' Confederation
Cement Admixtures Association
Cement and Concrete Association
Cement Makers' Federation
Concrete Society
Department of the Environment (Building Research Establishment)
Department of the Environment (Housing and Construction Industries)
Department of the Environment (Property Services Agency)
District Surveyors' Association
Federation of Civil Engineering Contractors
Greater London Council
Incorporated Association of Architects and Surveyors
Institute of Clerks of Works of Great Britain Incorporated
Institution of Civil Engineers
Institution of Structural Engineers
Precast Flooring Federation
Royal Institute of British Architects
Sand and Gravel Association Limited

This British Standard, having been prepared under the direction of the Civil Engineering and Building Structures Standards Committee, was published under the authority of the Board of BSI and comes into effect on
31 December 1985

The following BSI references relate to the work on this standard:
Committee reference CSB/39
Draft (ref 84/14533) announced *BSI News* October 1985

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Amendments issued since publication

Amd. No.	Date of issue	Comments
5918	May 1989	Indicated by a sideline in the margin

Contents

Foreword

This Part of BS 8110 has been prepared under the direction of the Civil Engineering and Building Structures Standards Committee.

BS 8110 deals with the structural use of concrete. It is published in three Parts:

— *Part 1: Code of practice for design and construction*;

— *Part 2: Code of practice for special circumstances*;

— *Part 3: Design charts for singly reinforced beams, doubly reinforced beams and rectangular columns*.

The design charts in this Part of BS 8110 have been prepared in accordance with the assumptions laid down in Part 1, with the intention that they may be used as standard charts and so avoid duplication of effort by individual design offices.

This Part of BS 8110 reflects the change in the characteristic strength of available reinforcing steel and supersedes CP 110-2:1972 and CP 110-3:1972 which are both withdrawn.

A British Standard does not purport to include all the necessary provisions of a contract. Users of British Standards are responsible for their correct application.

Compliance with a British Standard does not of itself confer immunity from legal obligations.

Summary of pages

This document comprises a front cover, an inside front cover, pages i to iv, pages 2 to 56, an inside back cover and a back cover.

This standard has been updated (see copyright date) and may have had amendments incorporated. This will be indicated in the amendment table on the inside front cover.

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1 General

1.1 Scope

This part of BS 8110 covers design charts for singly reinforced beams, doubly reinforced beams and rectangular columns. These design charts cannot be used to obtain the complete detailed design of any member but they may be used as an aid when analysing the cross section of a member at the ultimate limit state. The charts have been based on the assumptions laid down in BS 8110-1, use being made of the parabolic-rectangular stress block throughout. Full details of the derivation of the charts are given in Appendix.

NOTE The titles of the publications referred to in this standard are listed on the inside back cover.

1.2 Symbols

For the purposes of this Part of BS 8110 the definitions of the symbols given in BS 8110-1 and BS 8110-2 apply together with the following which are used in Appendix.

f_{sc} Stress in reinforcement in compression at the ultimate limit state

f_{st} Stress in reinforcement in tension at the ultimate limit state

ϵ_o Strain at the point on the parabolic-rectangular stress-strain diagram where the parabolic section joins the linear section ($\epsilon_o = 2.4 \times 10^{-4} \sqrt{\{f_{cu}/\gamma_m\}}$).

1.3 Use of charts

Design examples illustrating the use of the charts are given in Appendix B.

The values of K indicated on the column design charts are the additional moment reduction coefficients given by equation 33 of BS 8110-1:1985.

2 Design charts

Design charts Nos. 1 to 50 are given on the following pages.

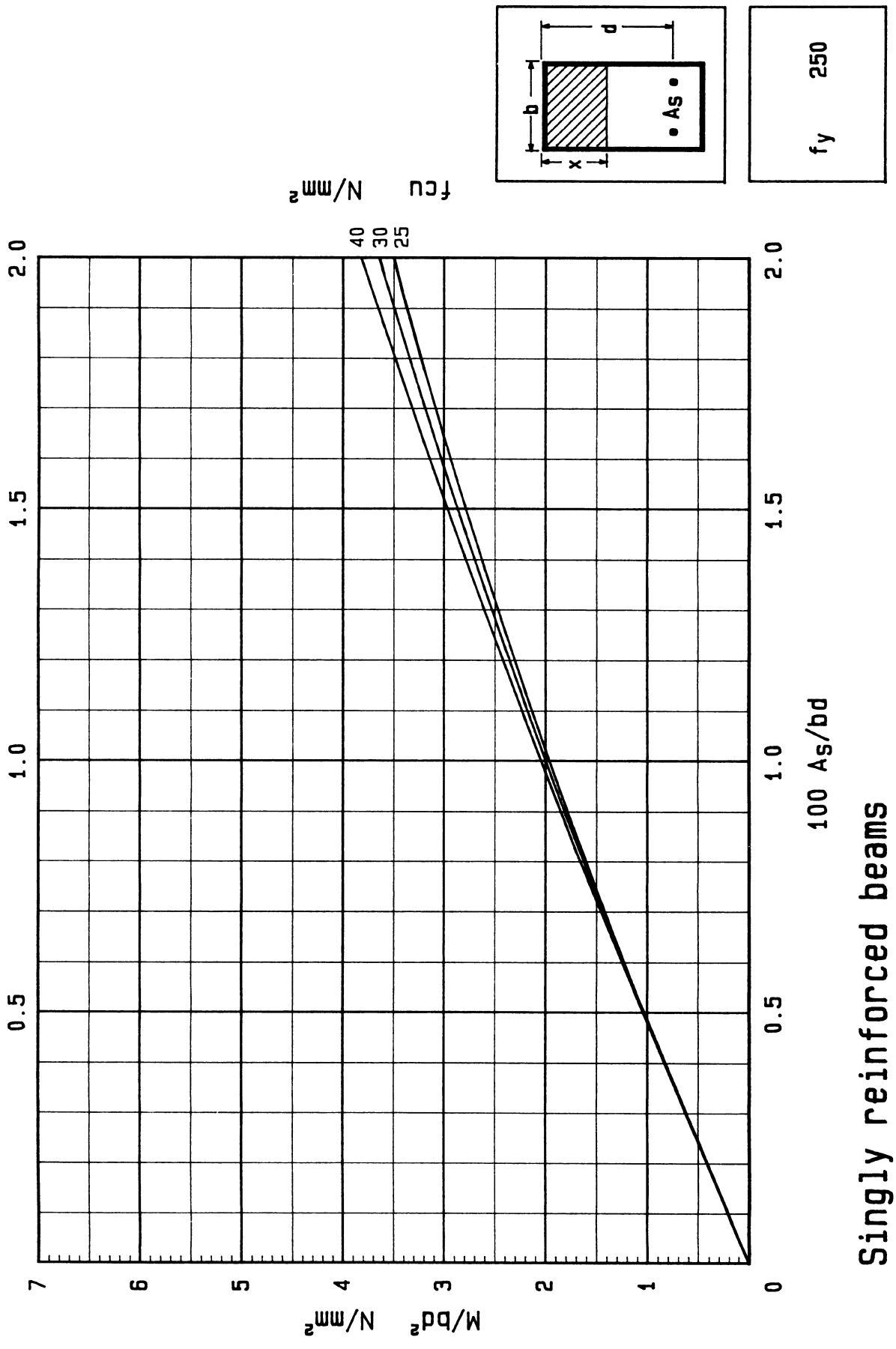
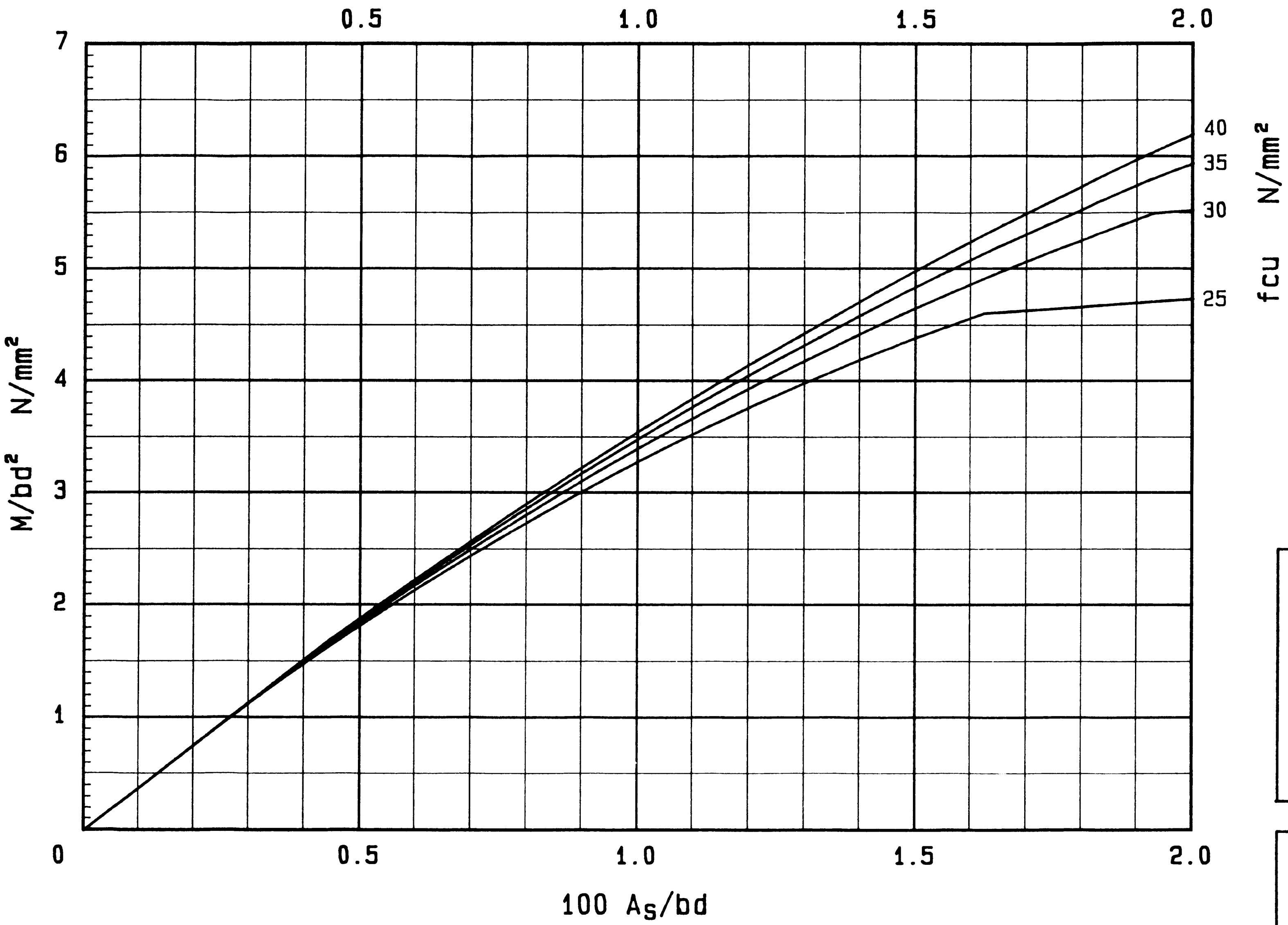
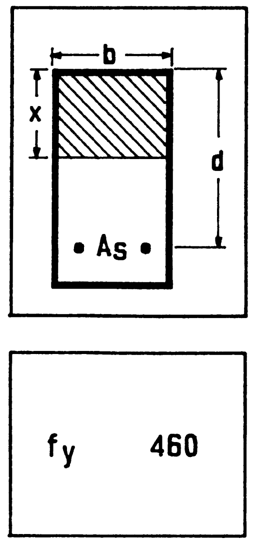


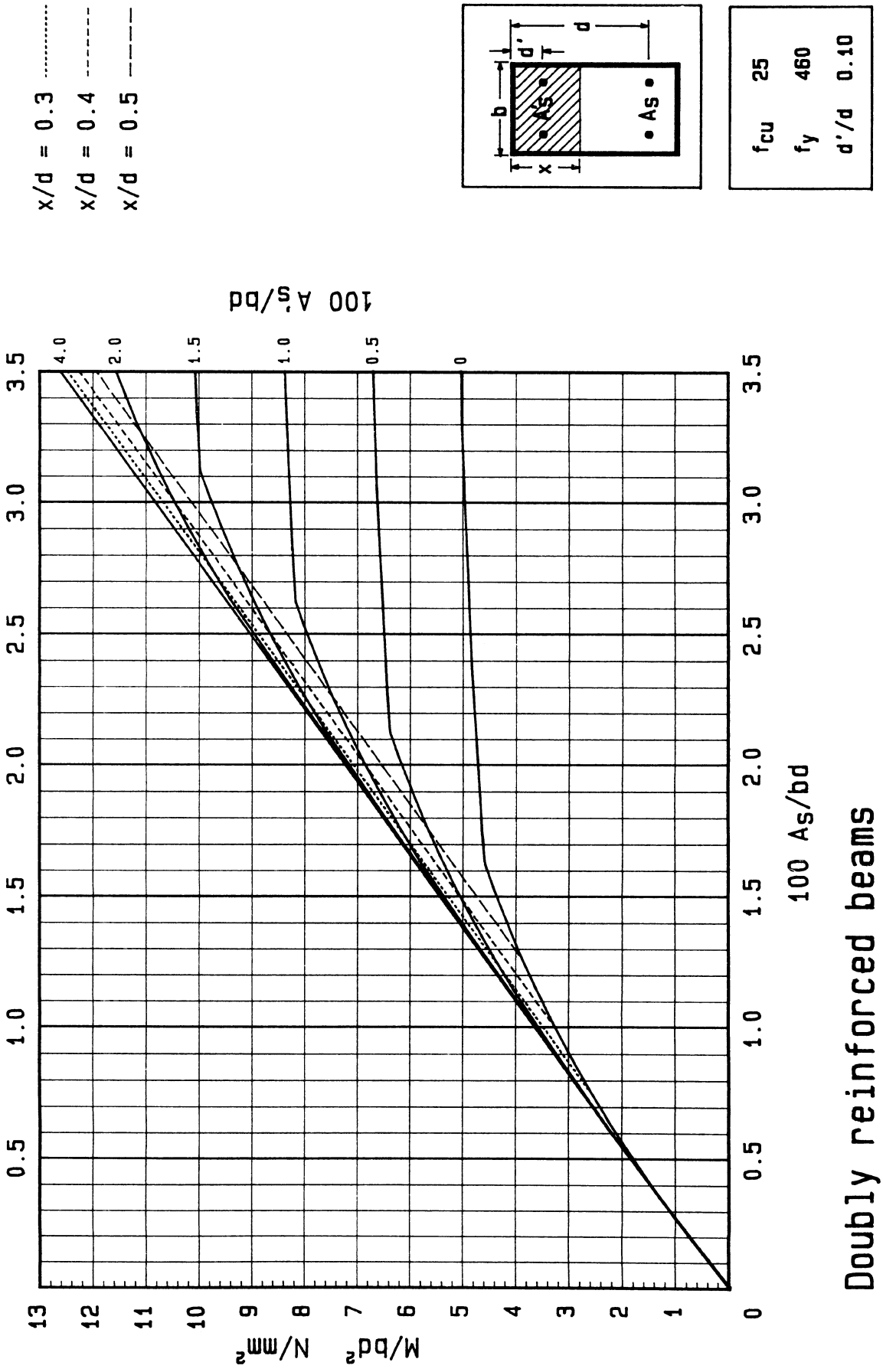
Chart No. 1

Chart No. 2





Singly reinforced beams



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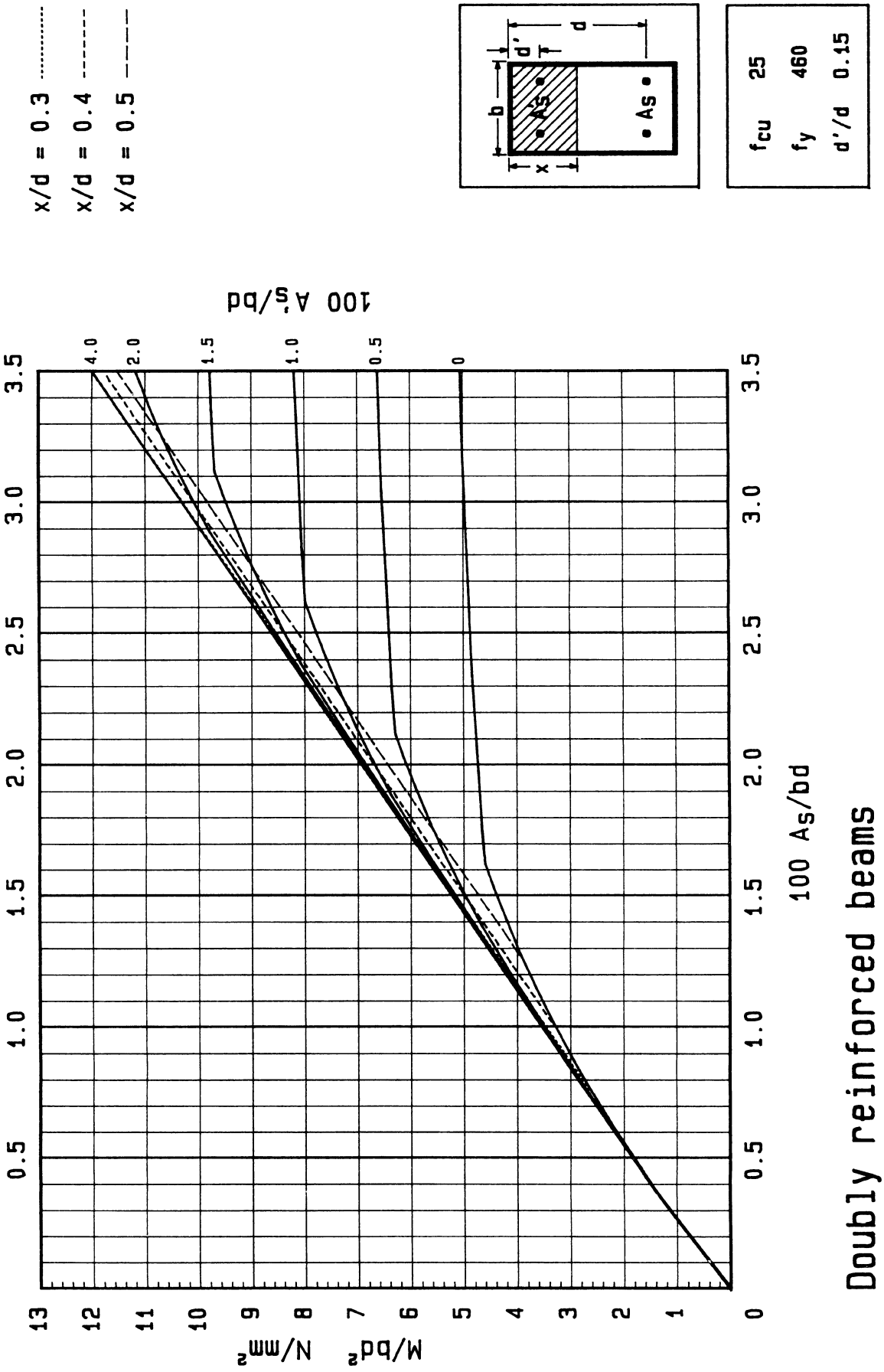

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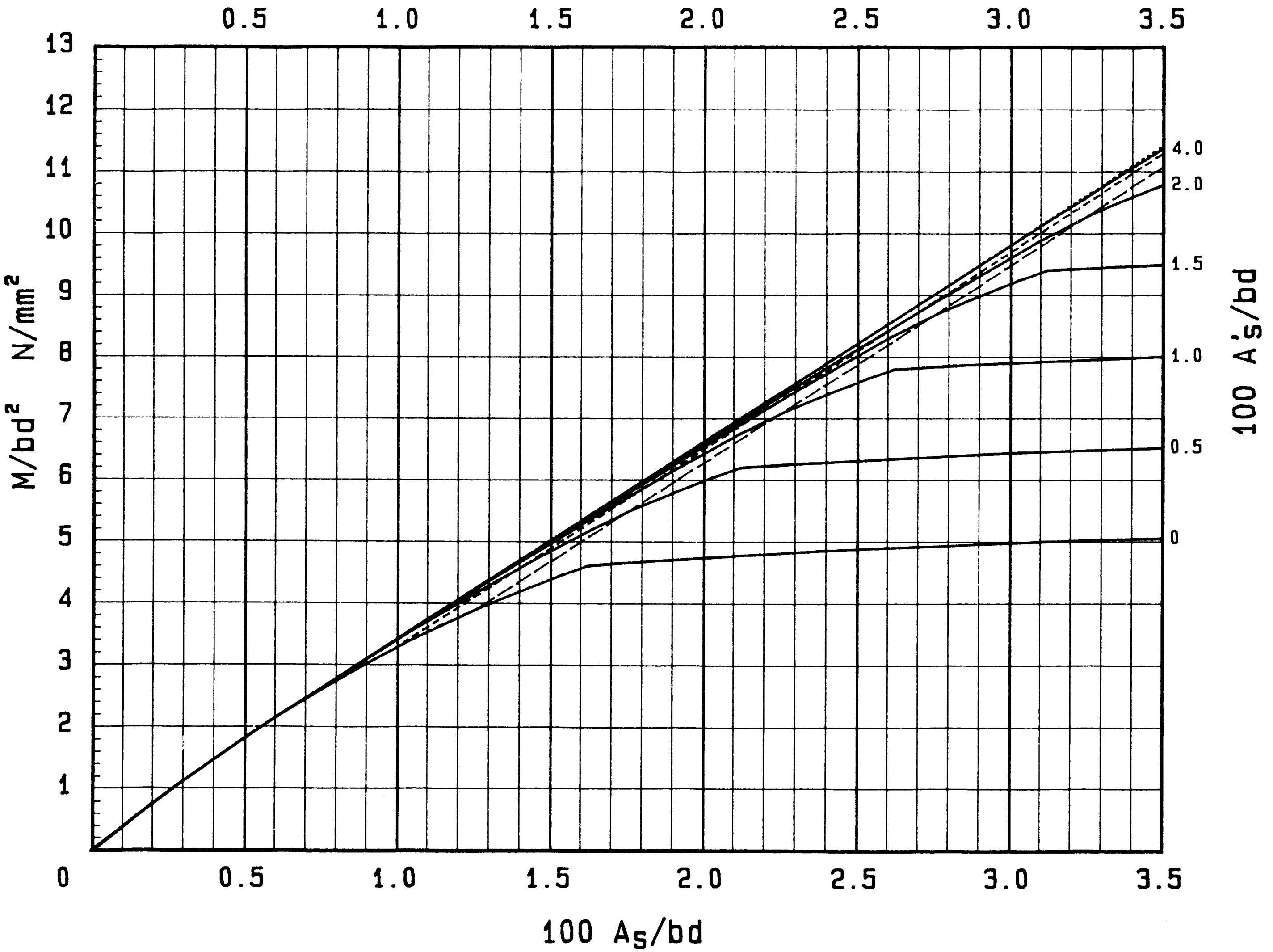

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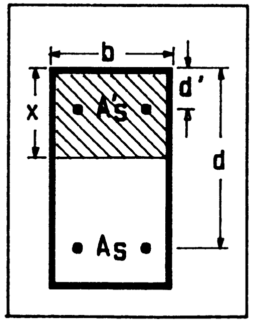
x/d = 0.4 - - - - -
x/d = 0.5 — — —

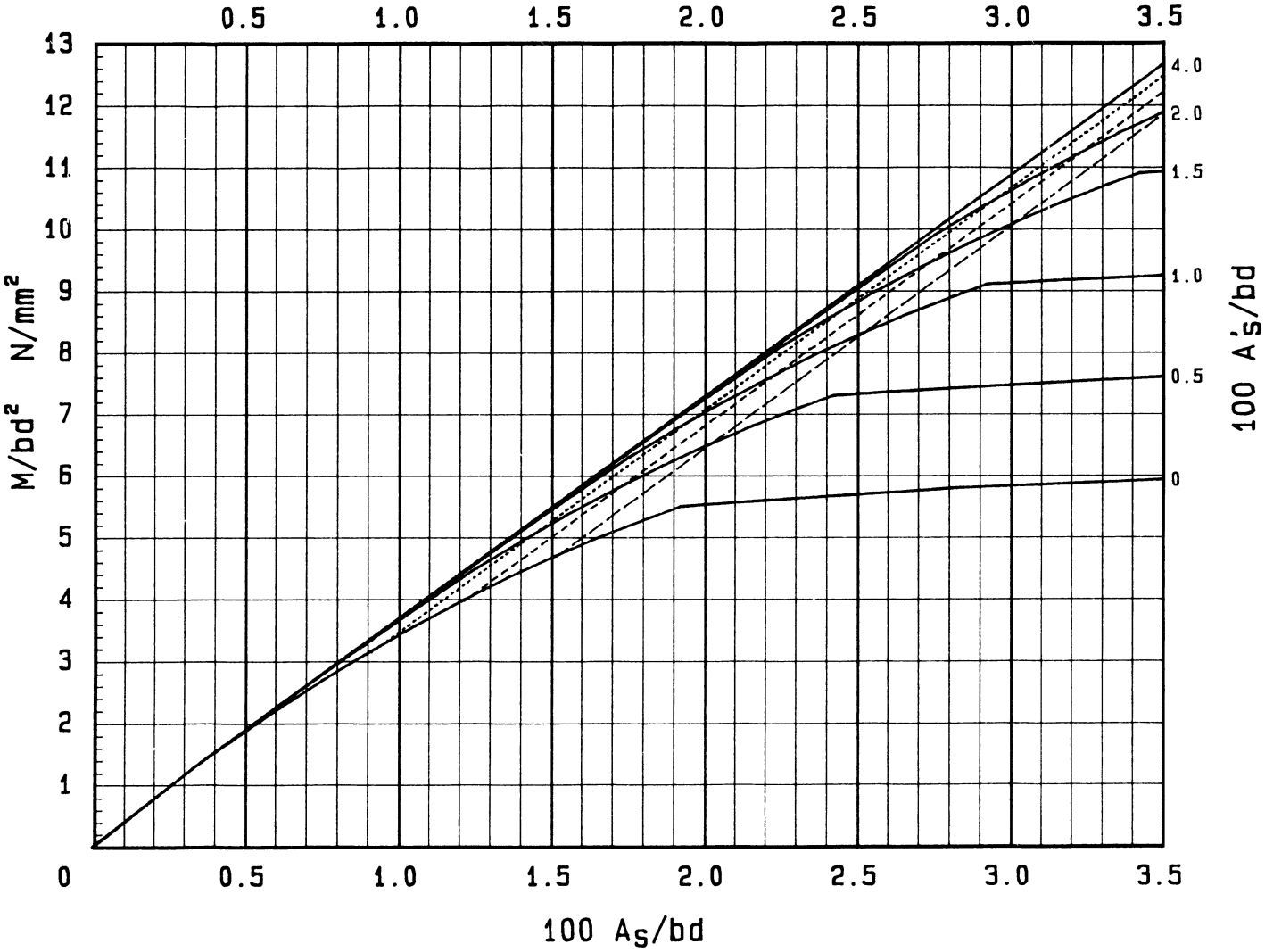


f_{cu}	25
f_y	460
d'/d	0.20

Chart No. 5

Doubly reinforced beams

Chart No.6

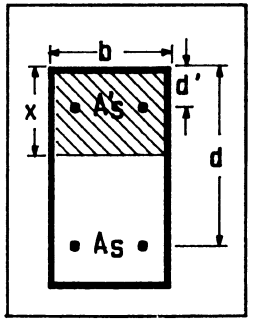


Doubly reinforced beams

x/d = 0.3 .....

x/d = 0.4 -------

x/d = 0.5 — · — · —



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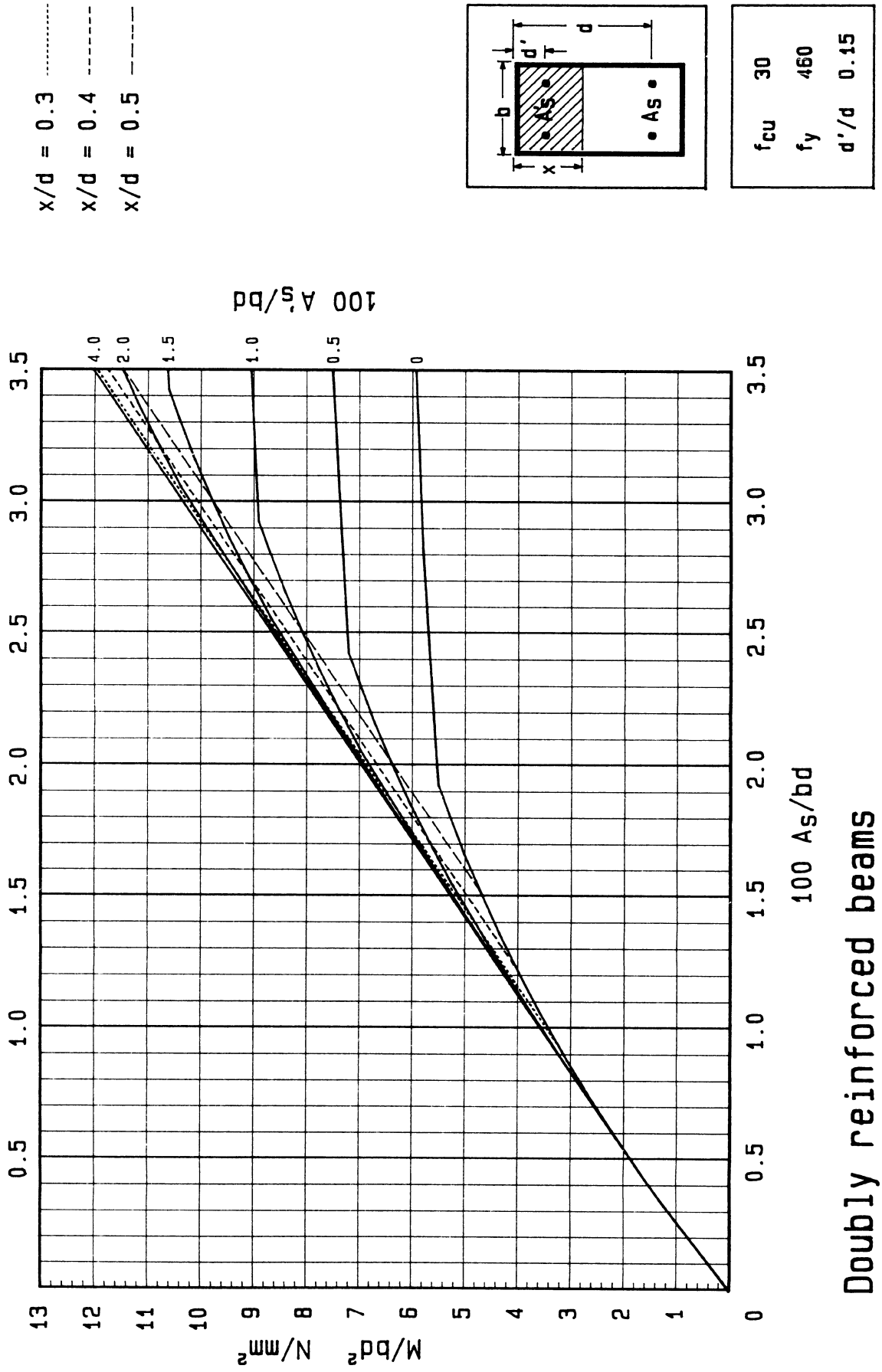
f_y	460
d'/d	0.10



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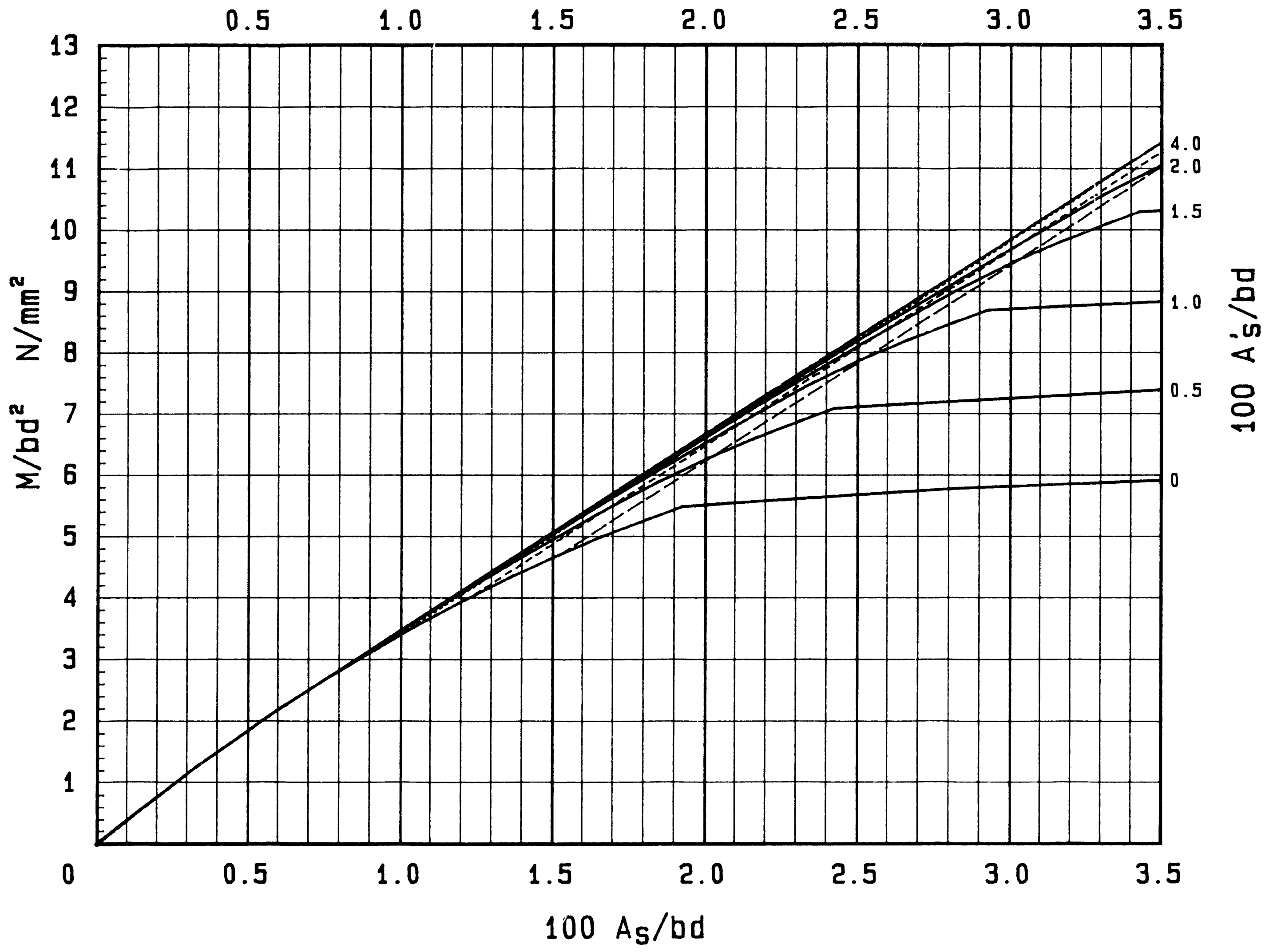
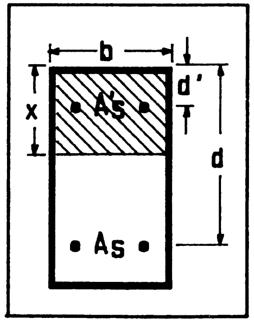


x/d = 0.3 ....

x/d = 0.4 -------

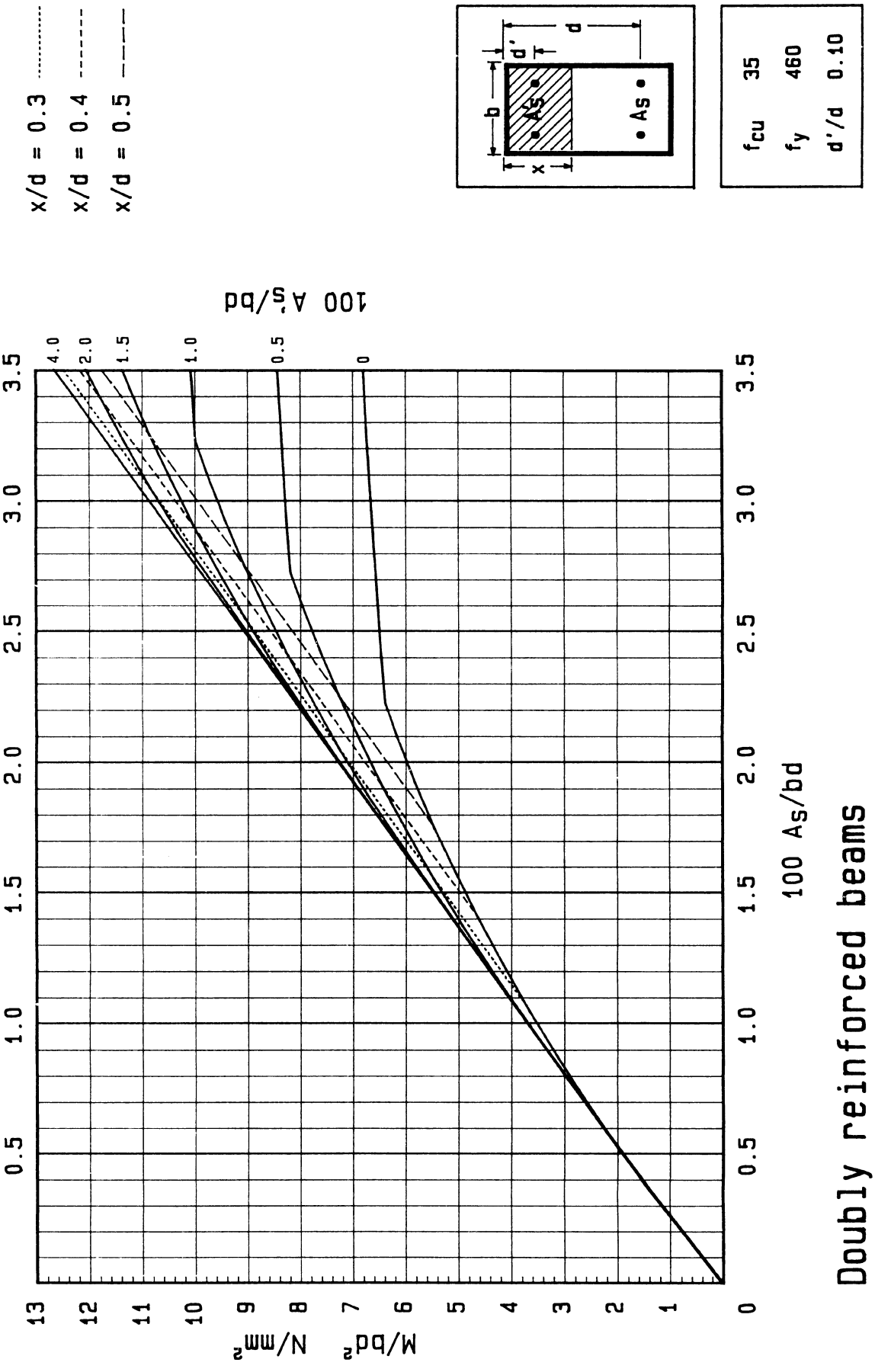
x/d = 0.5 ––––



f_{cu}	30
f_y	460
d'/d	0.20

Chart No. 8

Doubly reinforced beams



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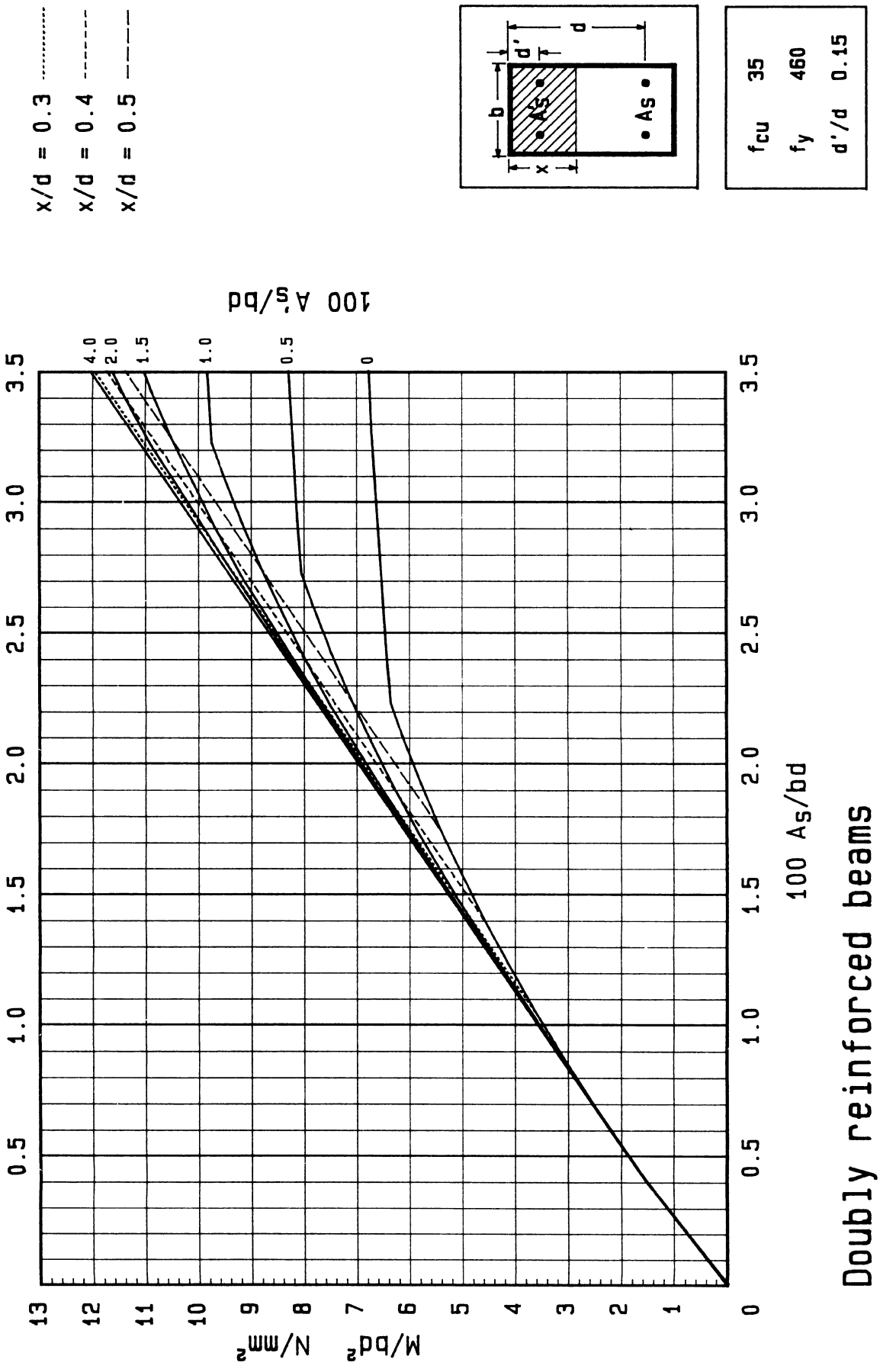

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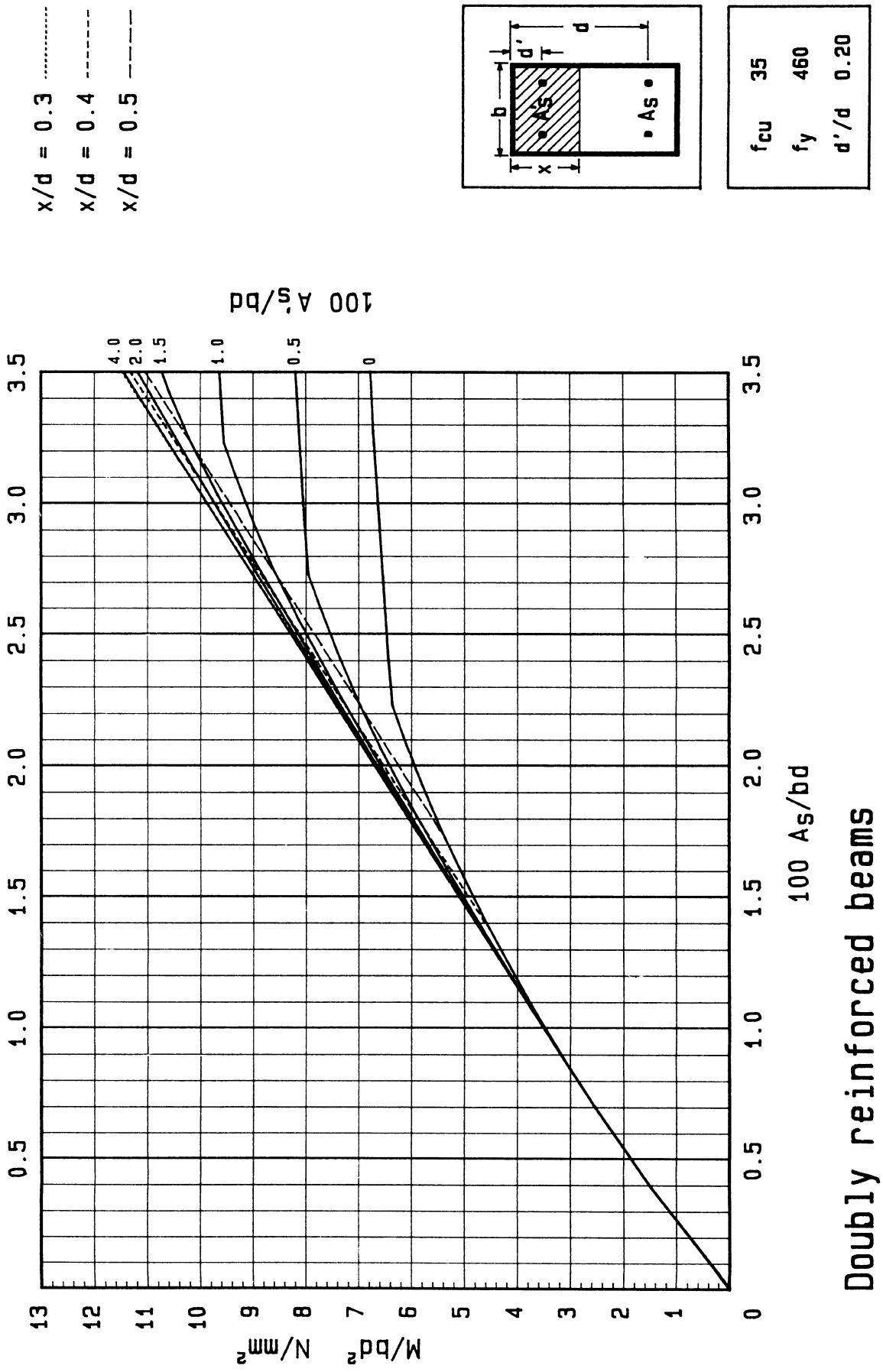

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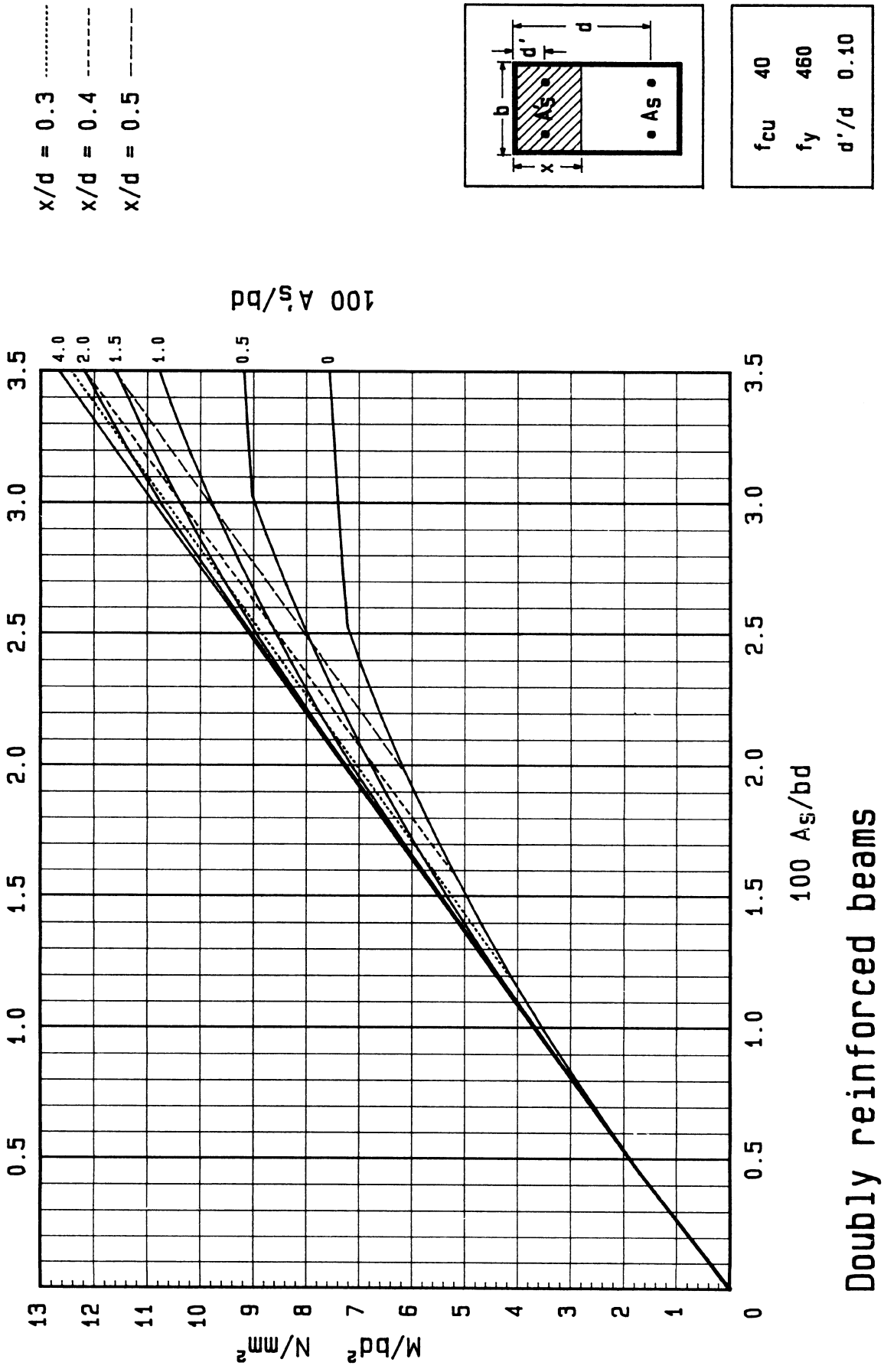

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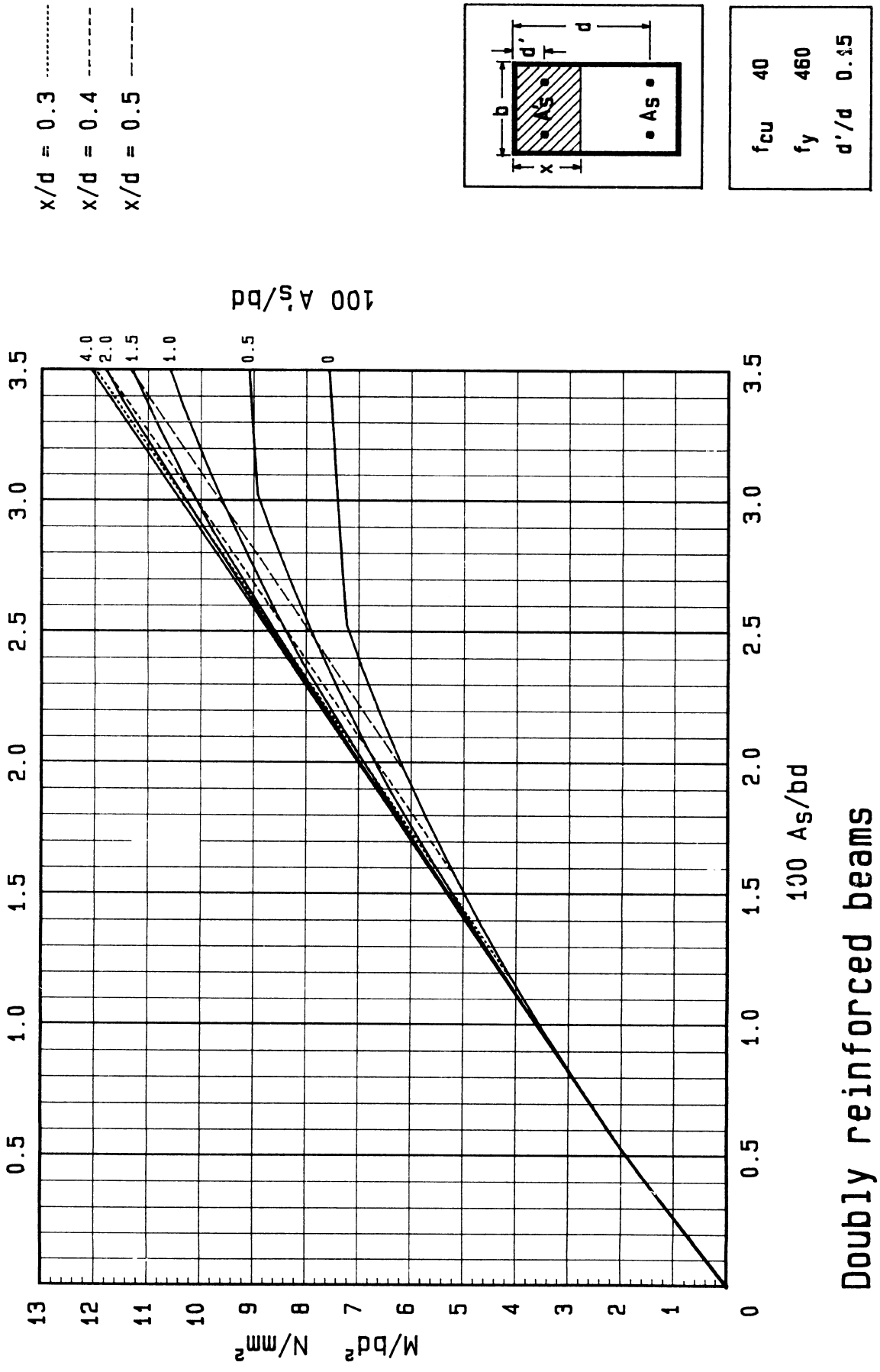

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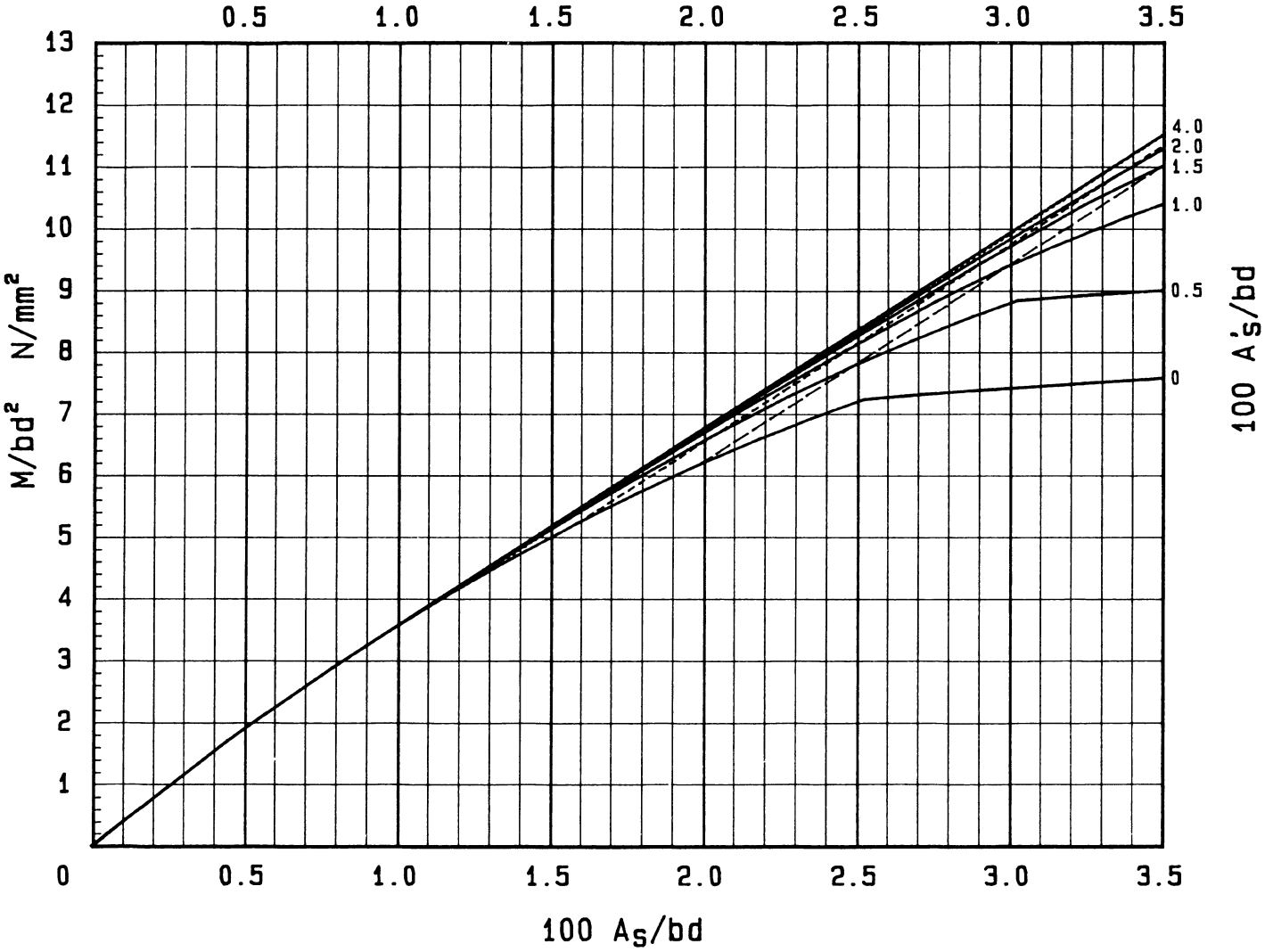
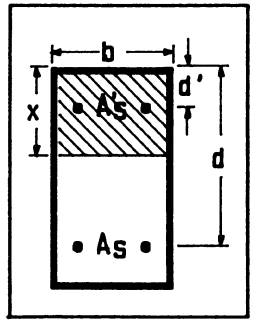
Chart No. 14



x/d = 0.3 .....

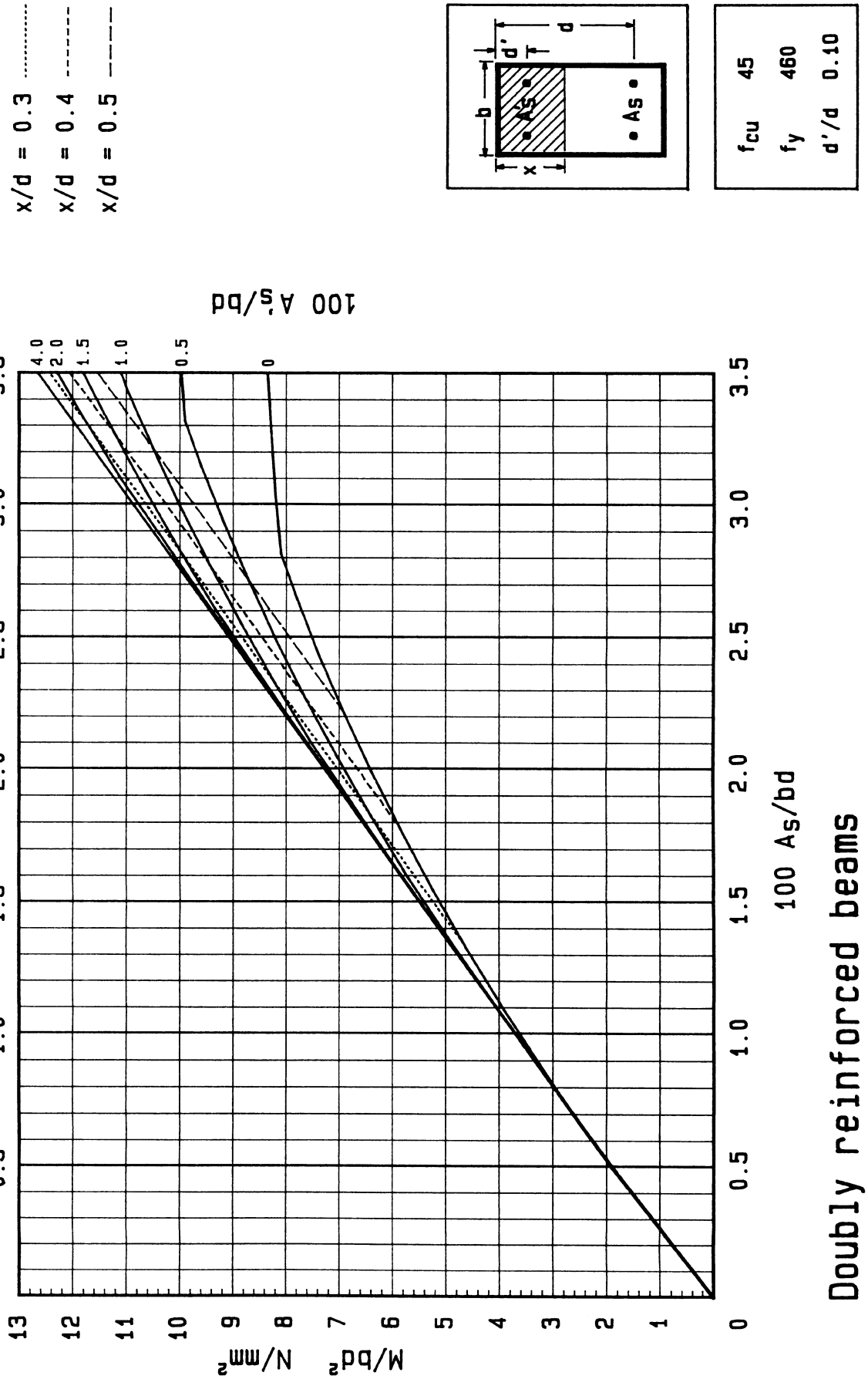
x/d = 0.4 -------

x/d = 0.5 — — —



f_{cu}	40
f_y	460
d'/d	0.20

Doubly reinforced beams



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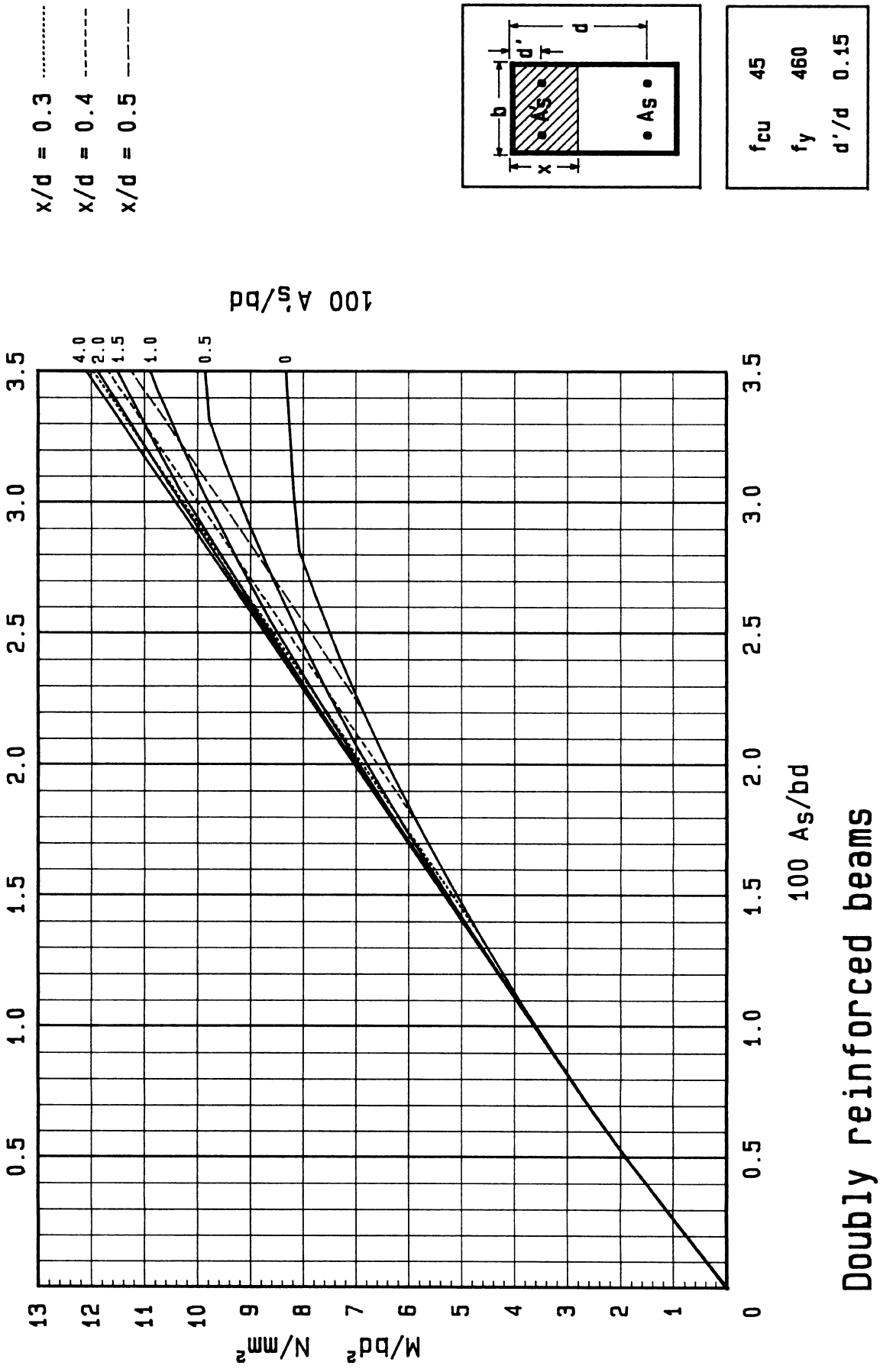

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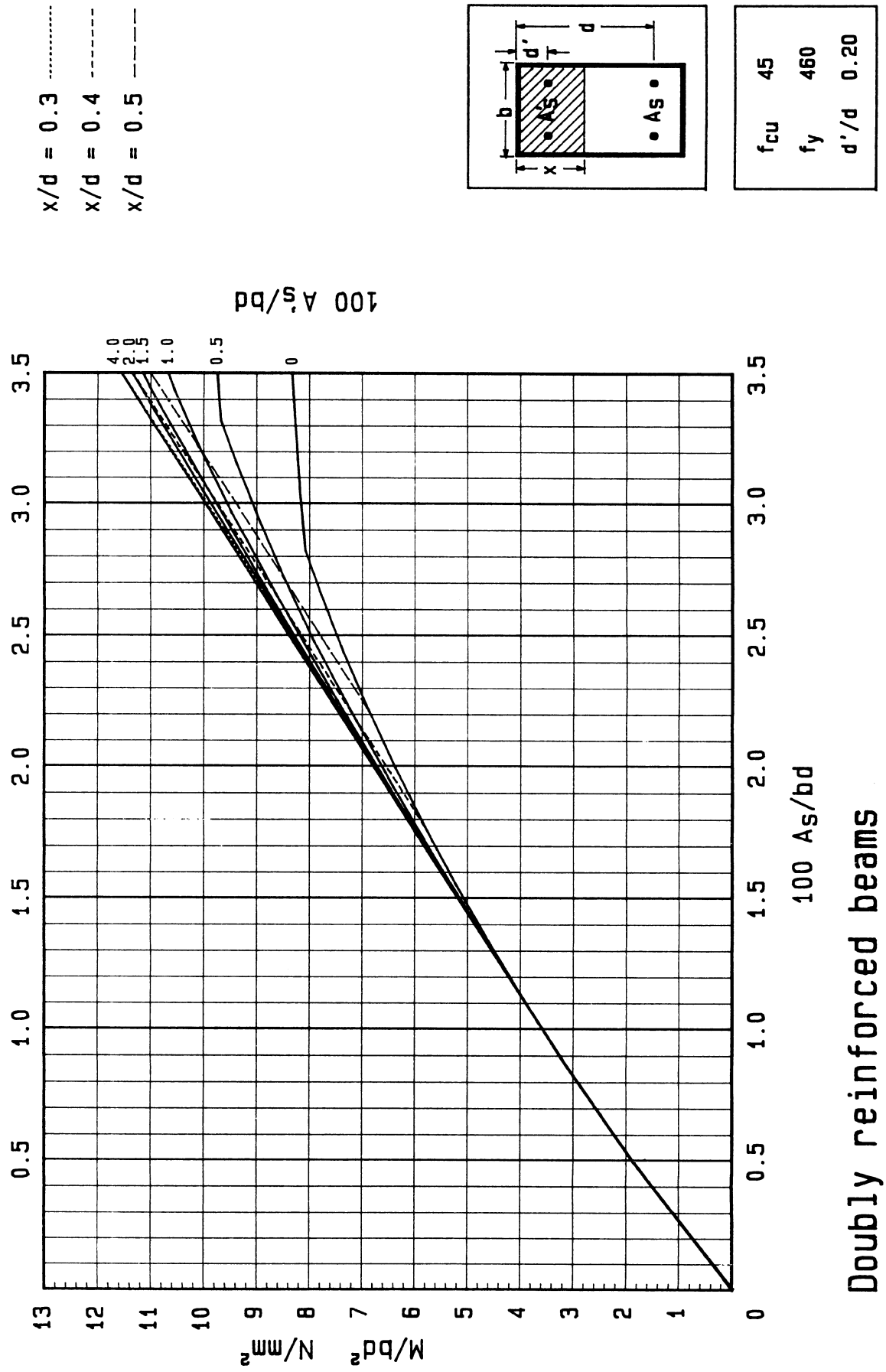

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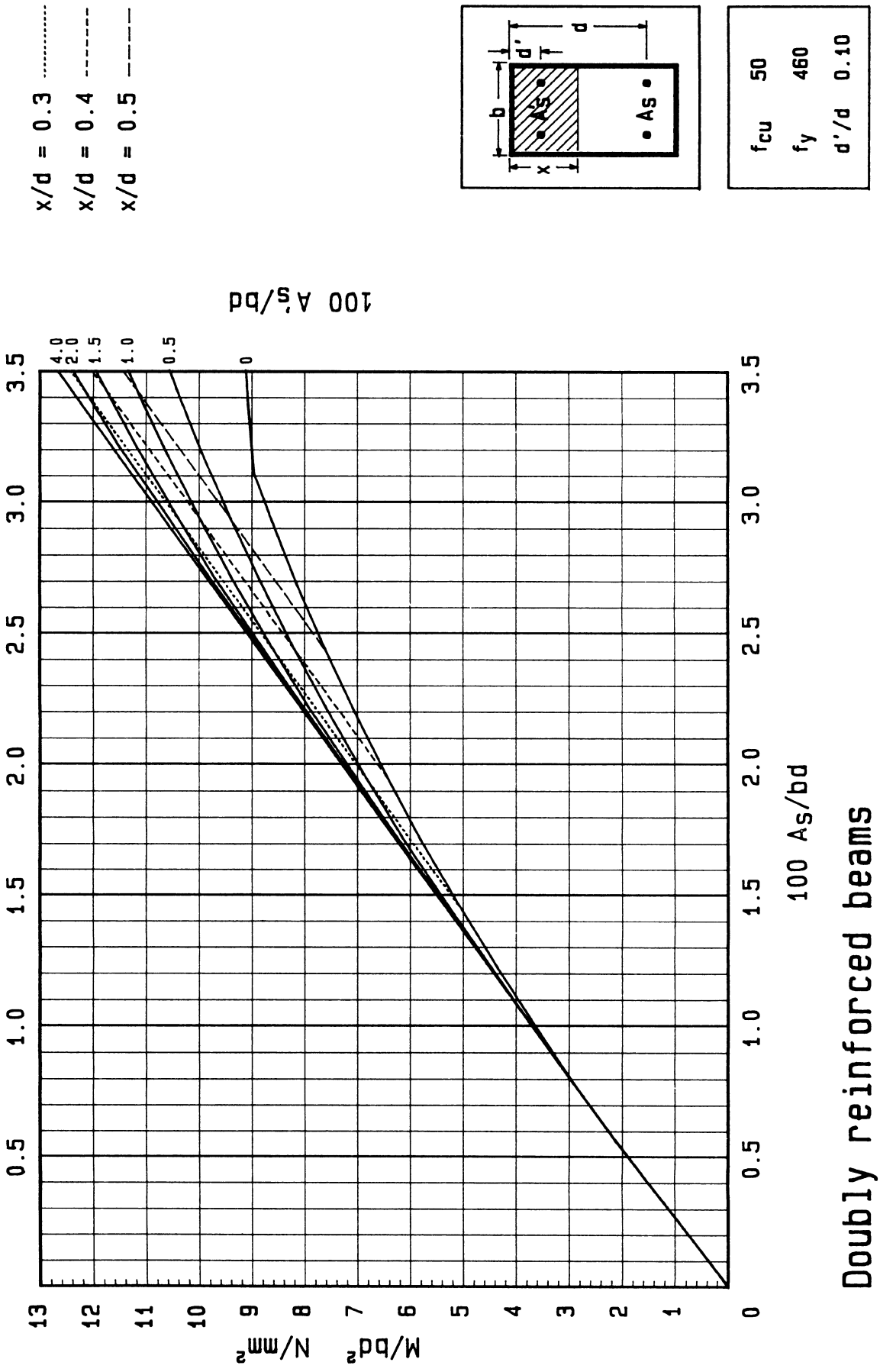

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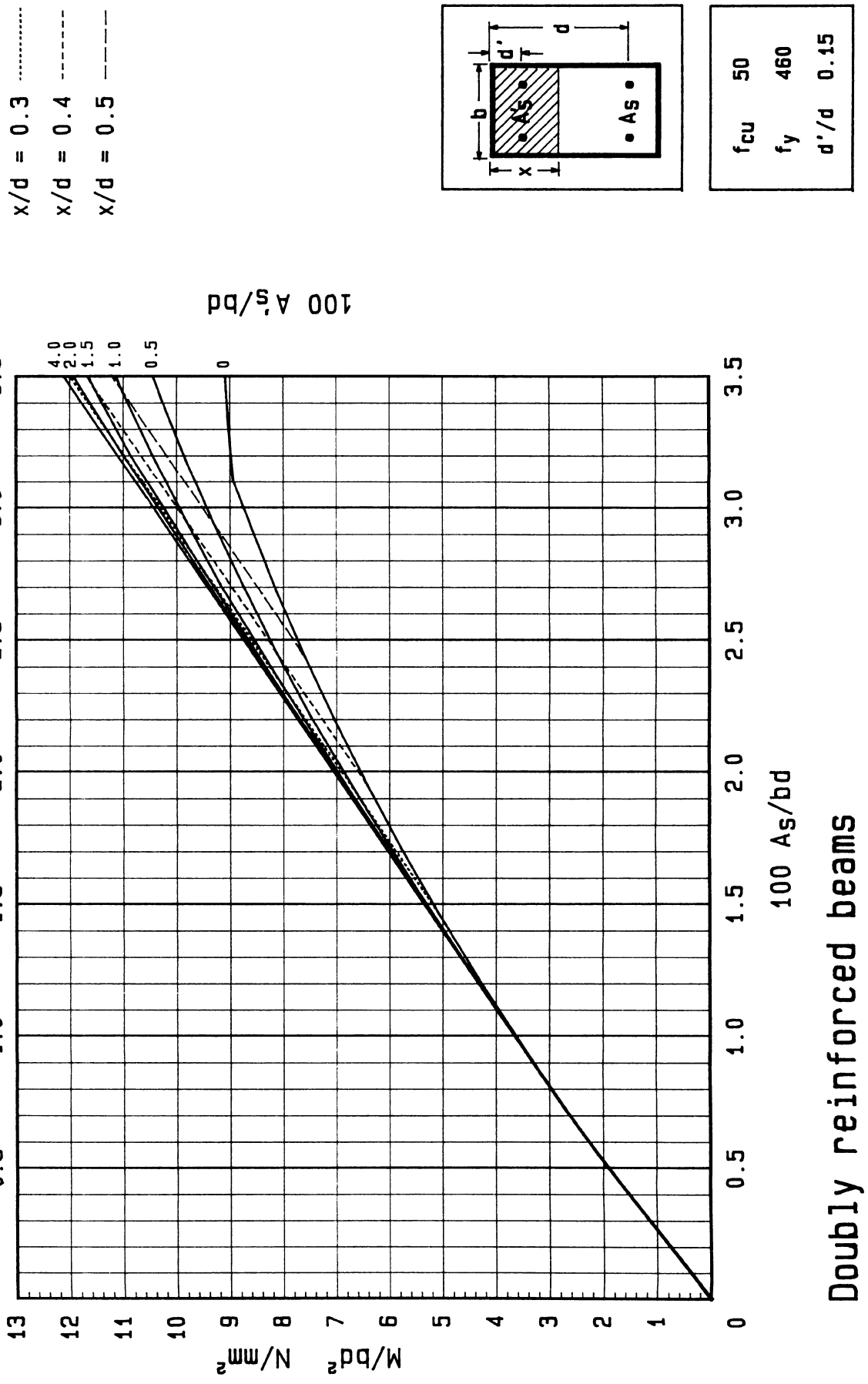

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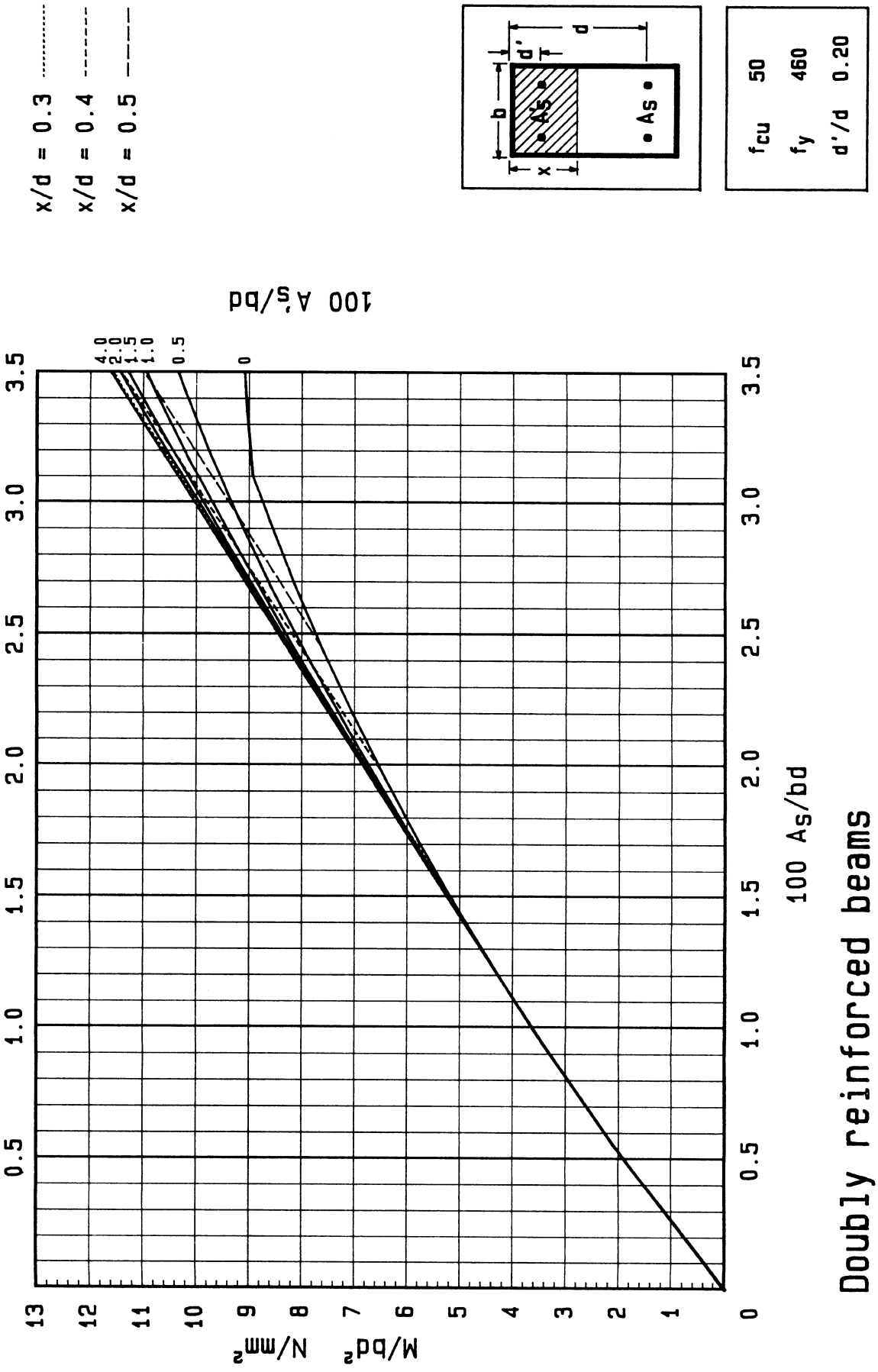

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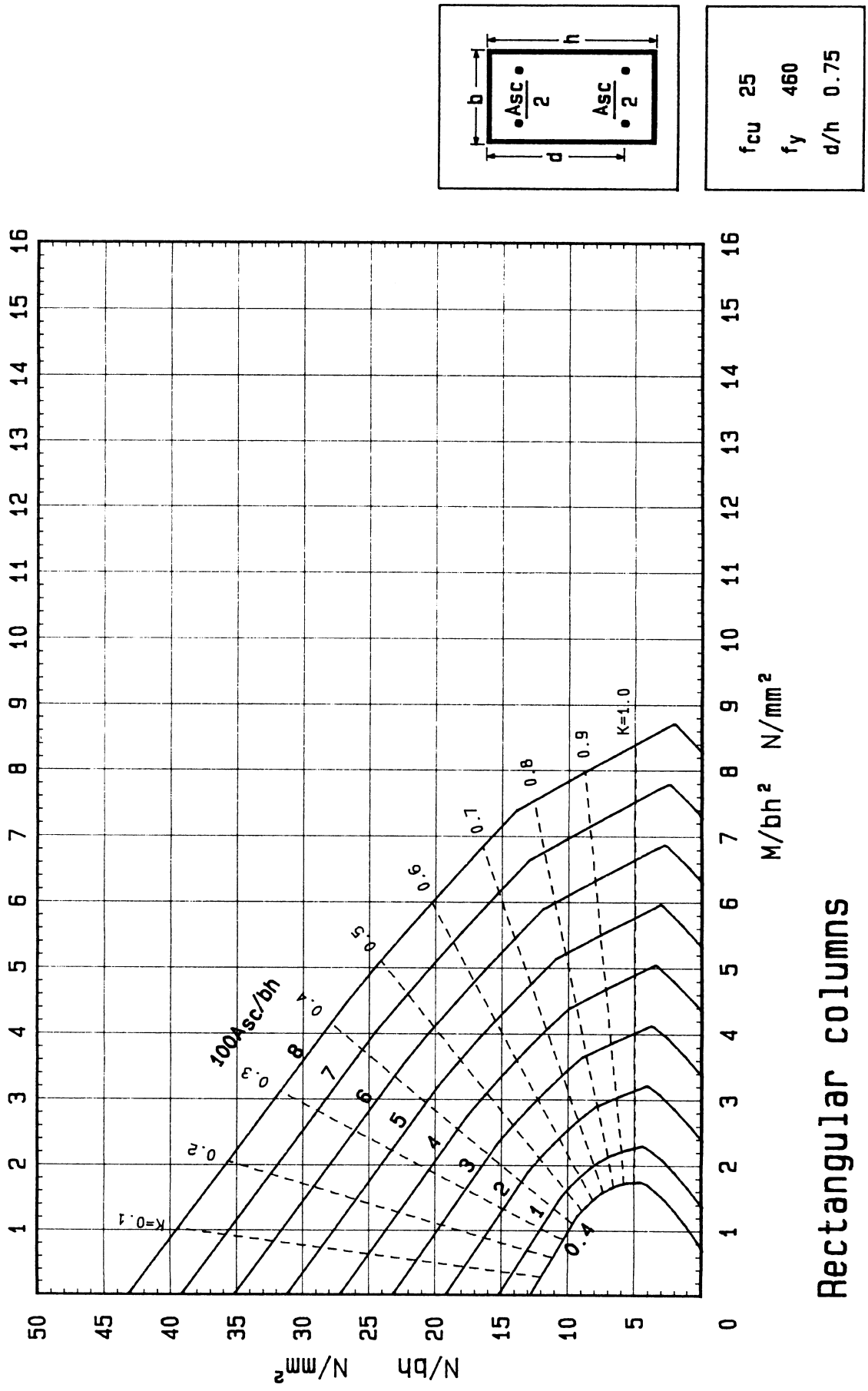

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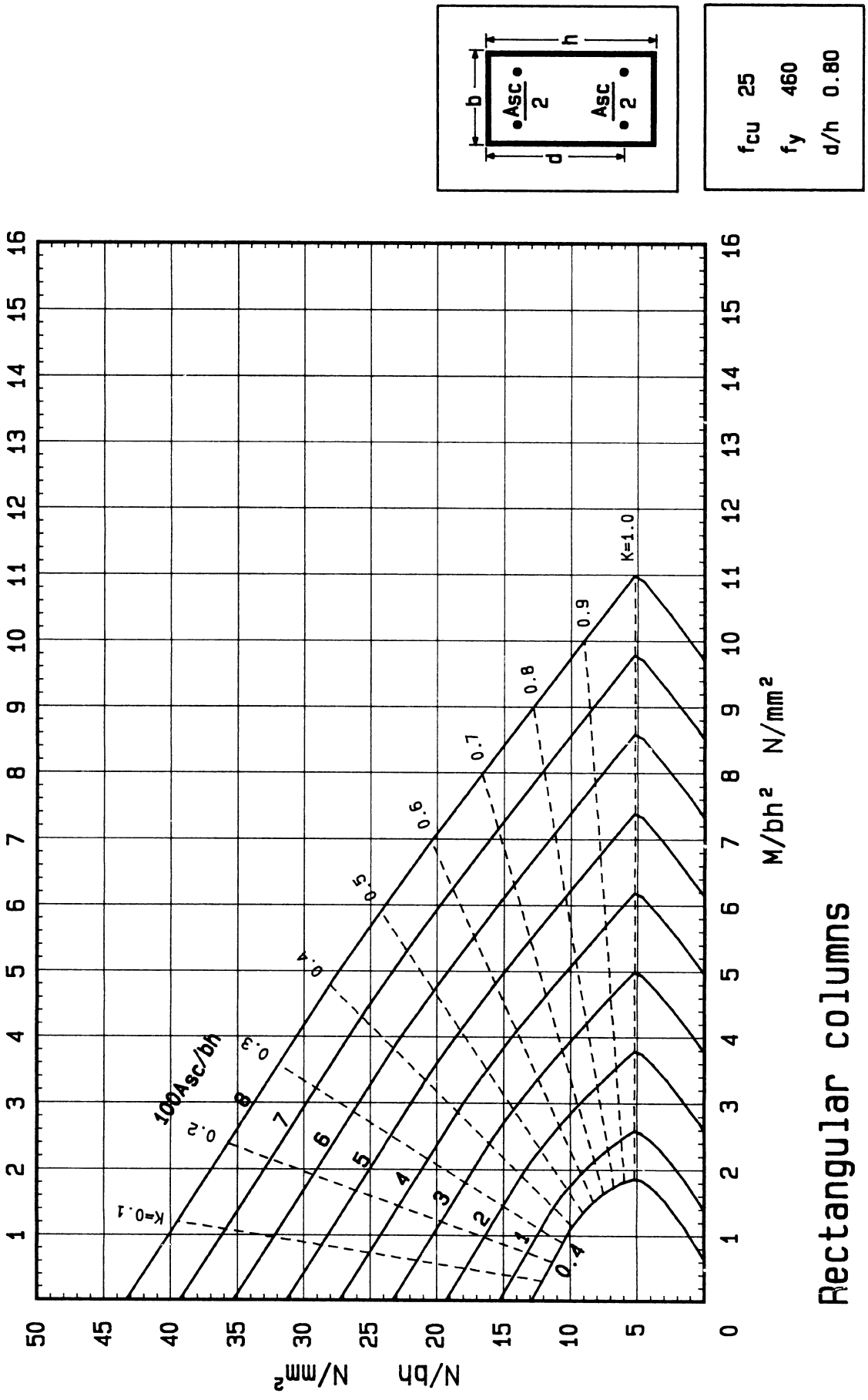
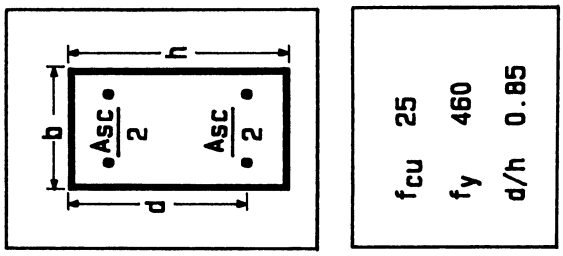
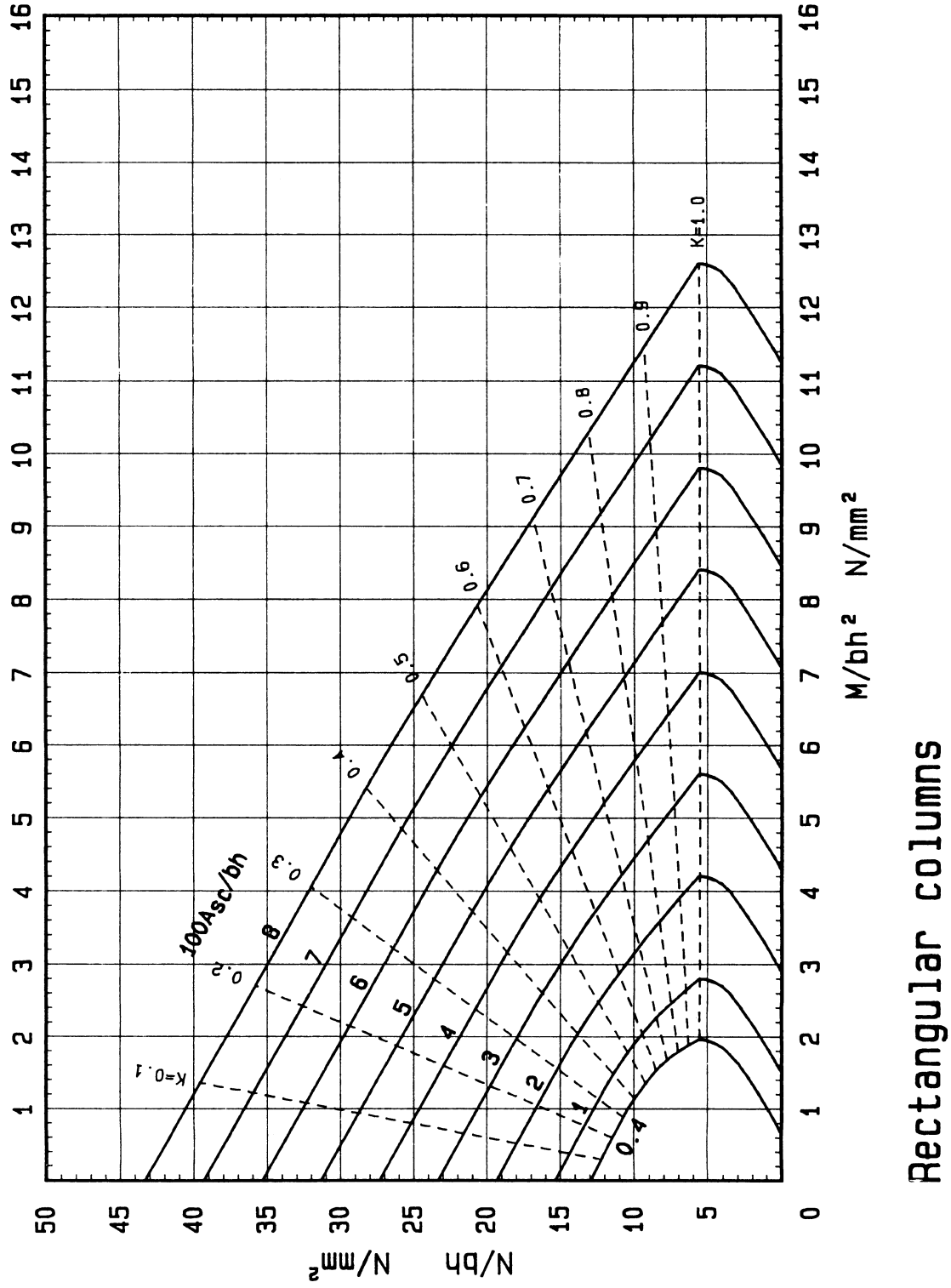




f_{cu} 25
f_y 460
d/h 0.85



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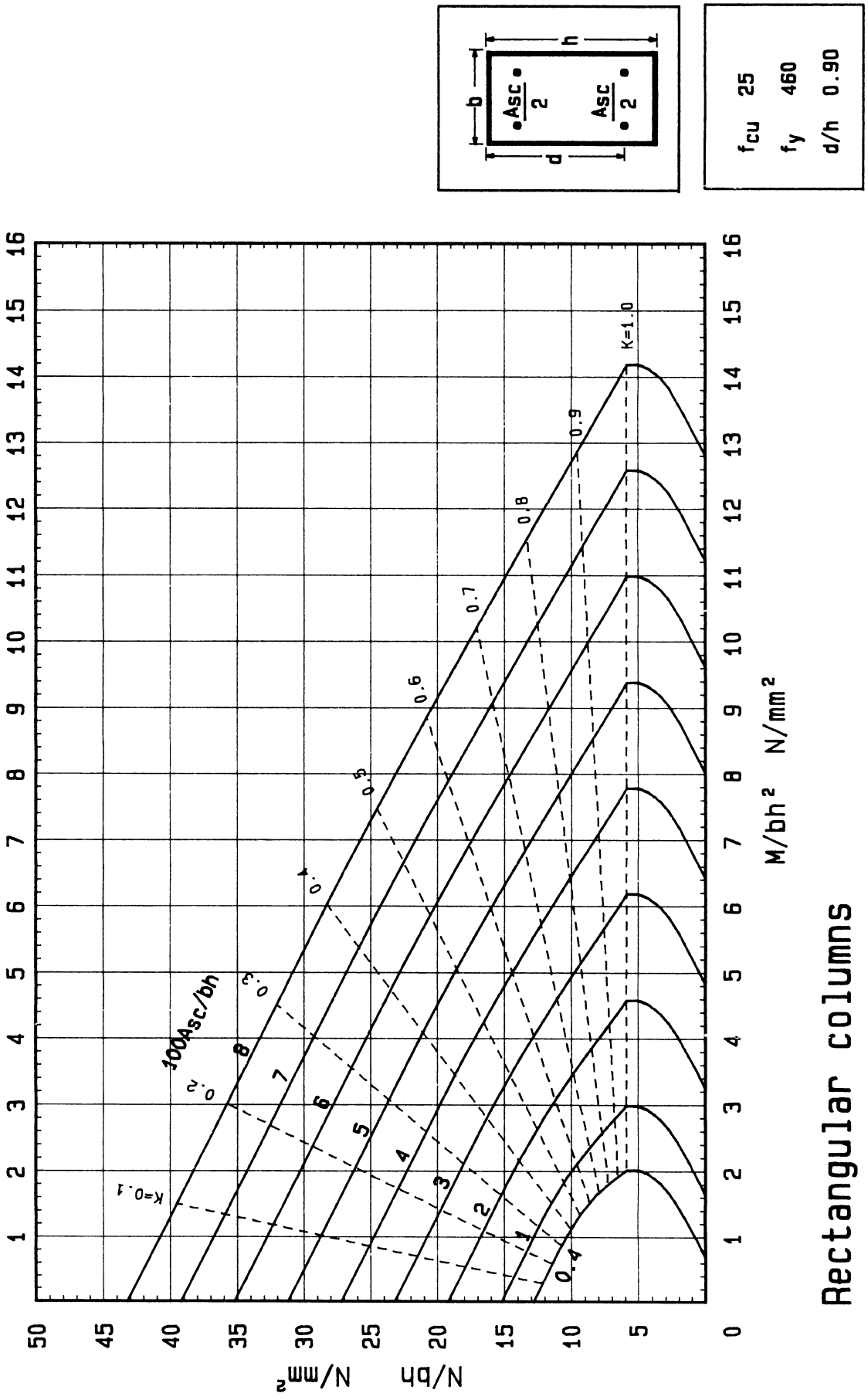

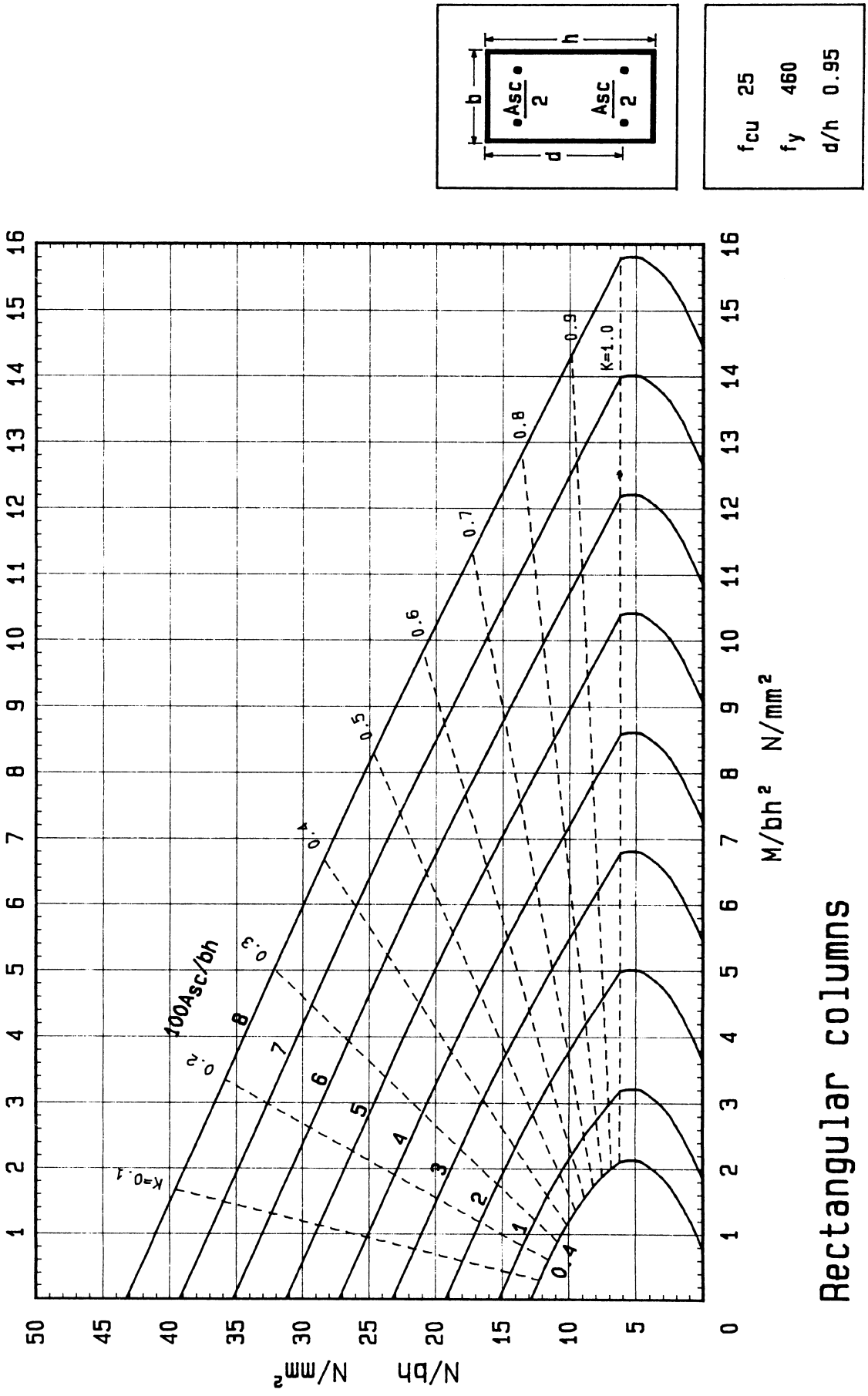

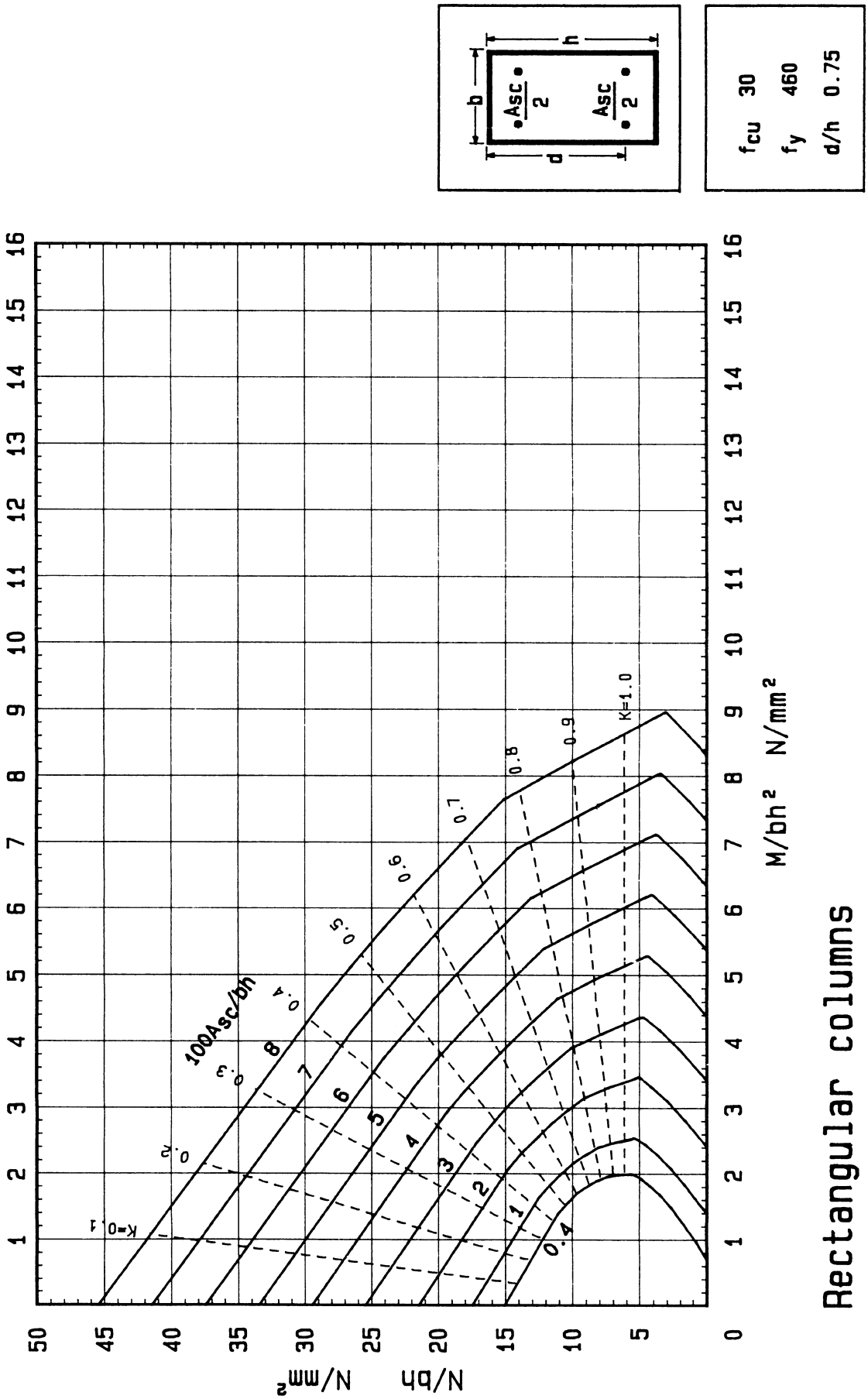

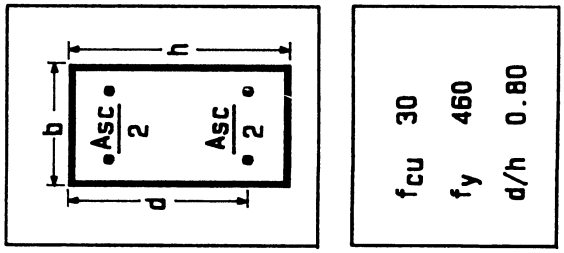

f_{cu} 30
f_y 460
d/h 0.80

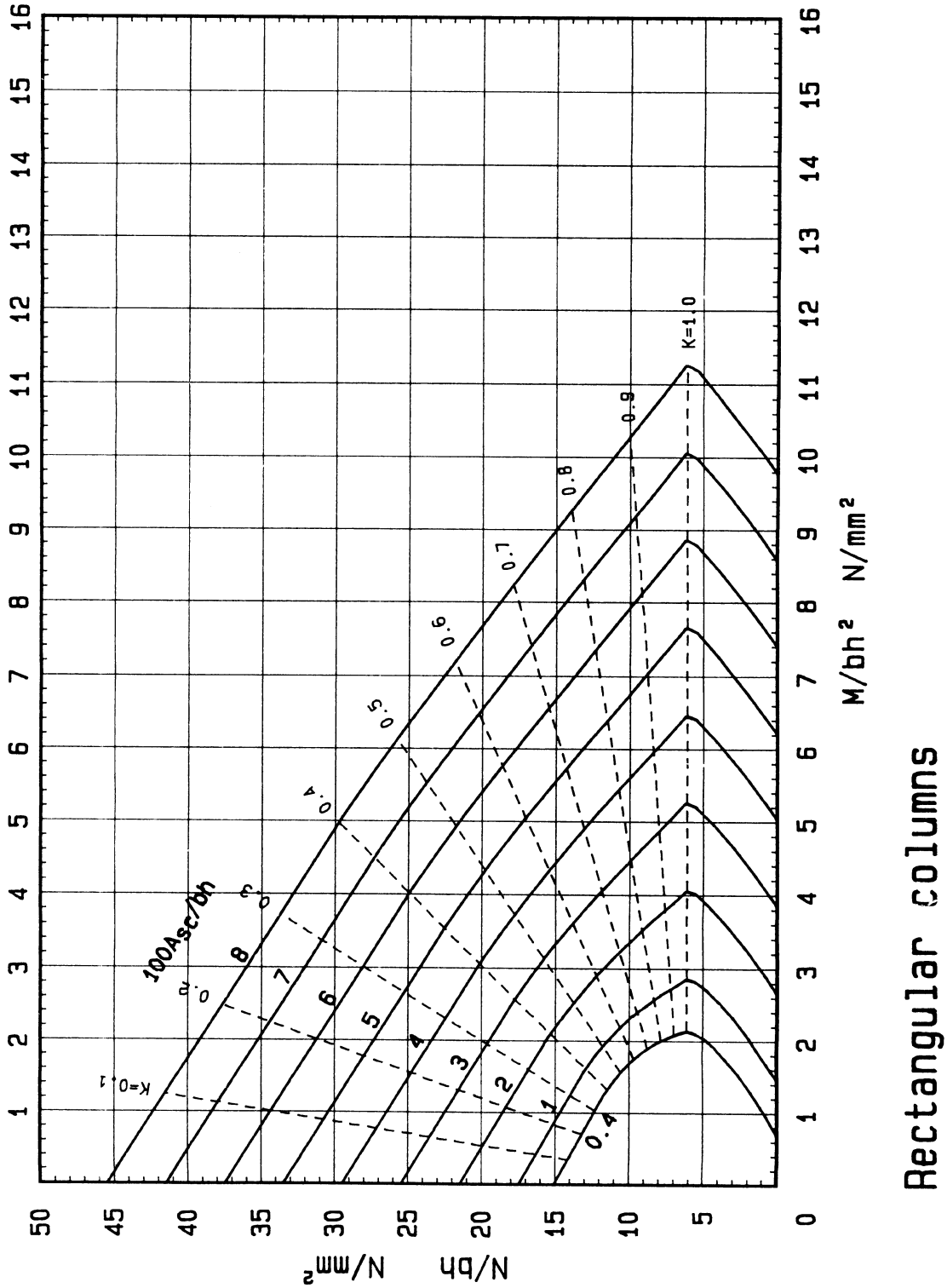

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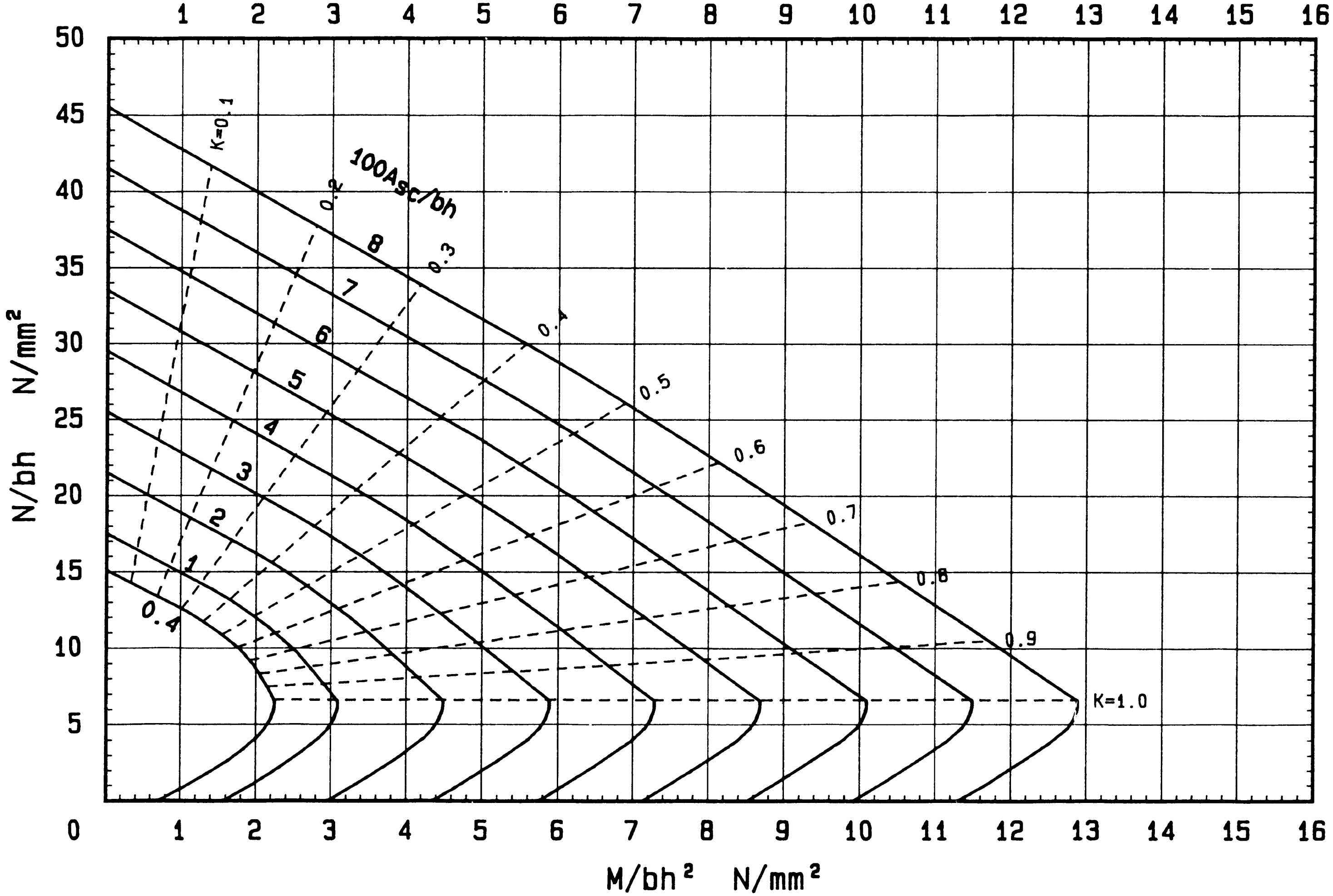
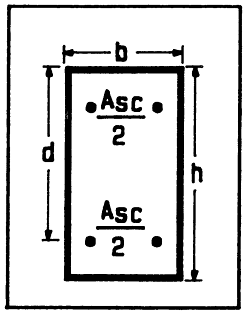
Chart No. 28

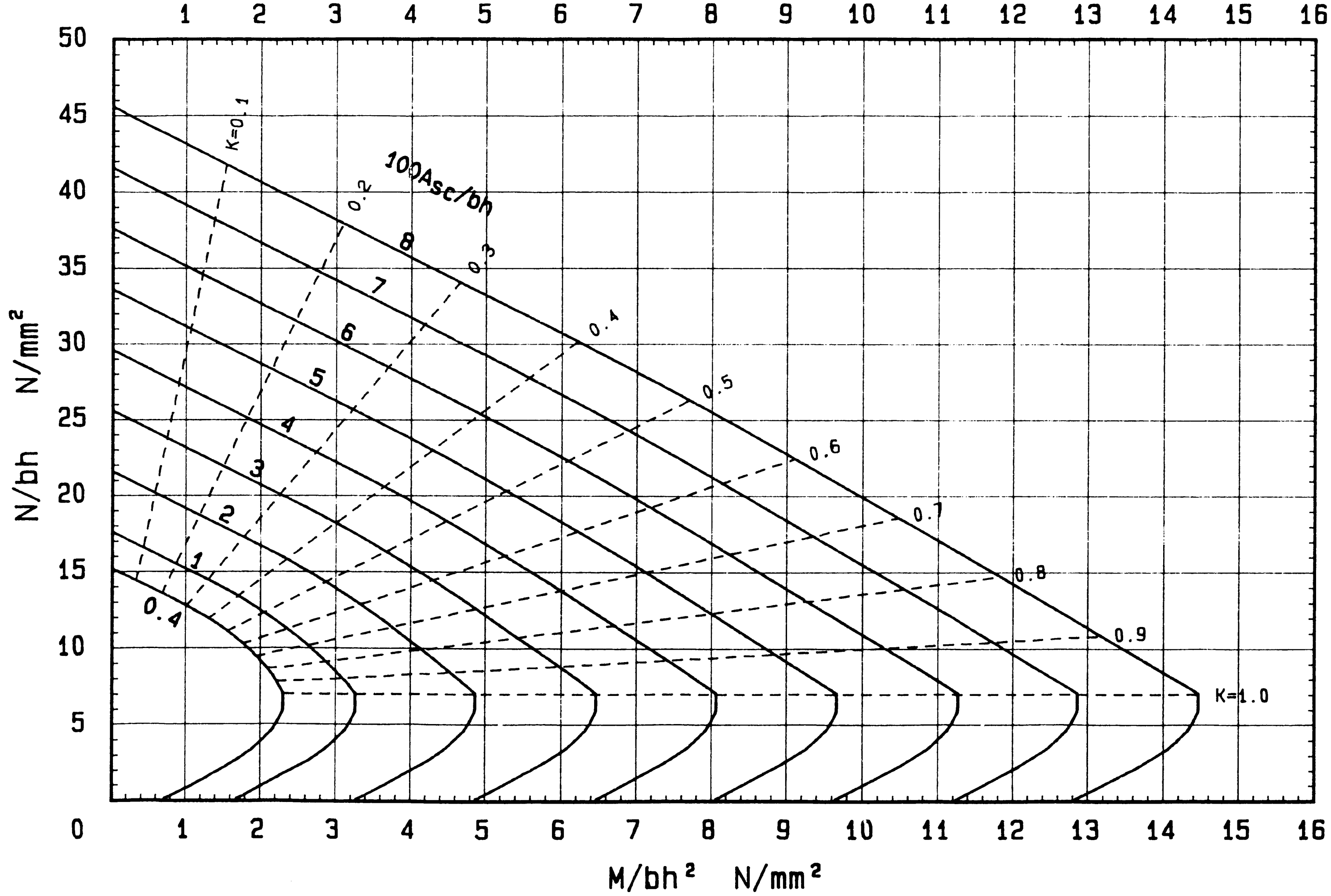


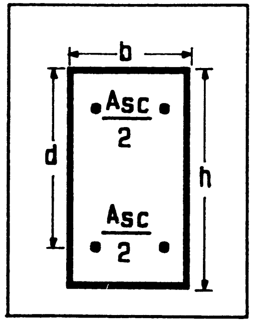


f_{cu}	30
f_y	460
d/h	0.85

Rectangular columns

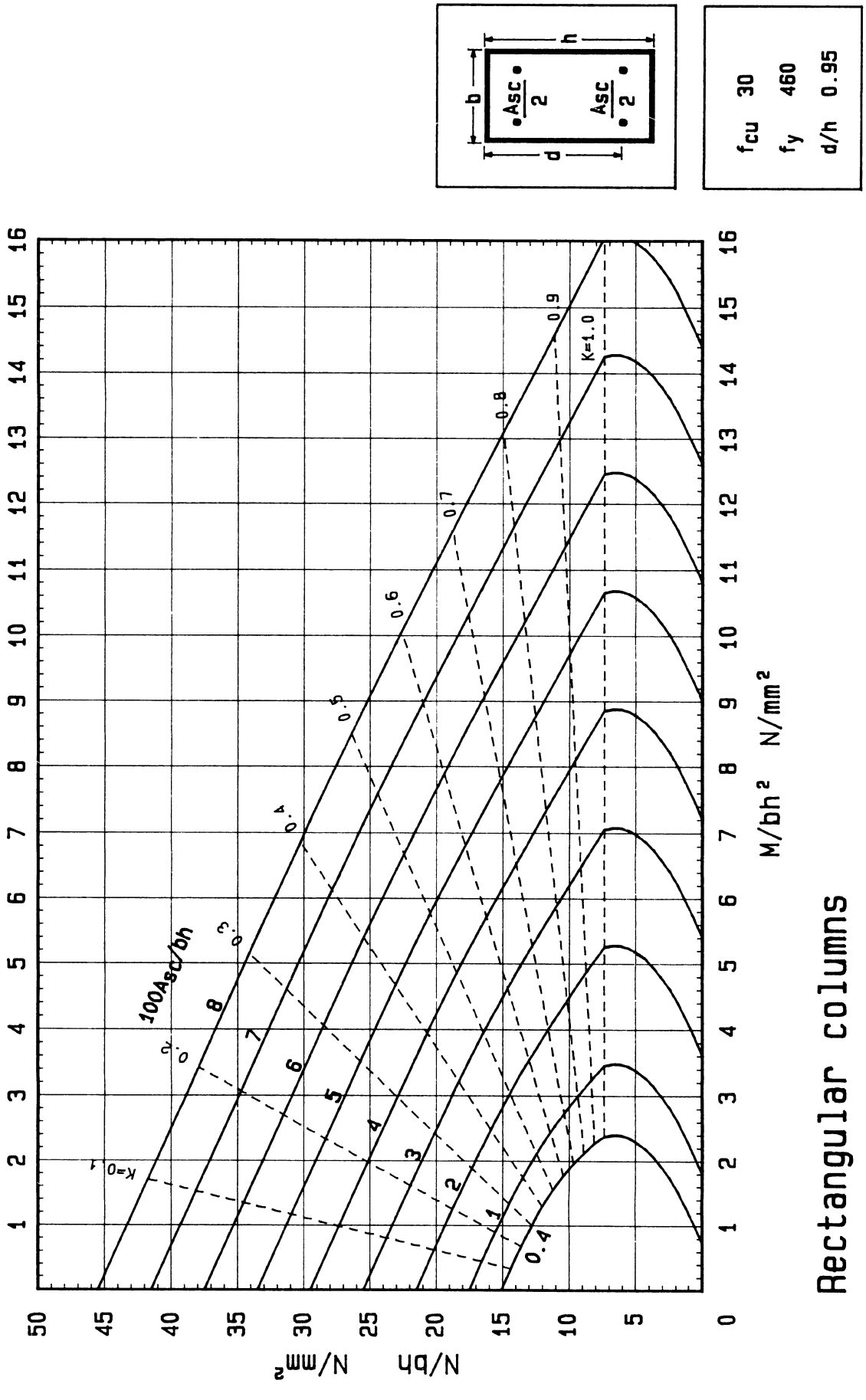
Chart No. 29

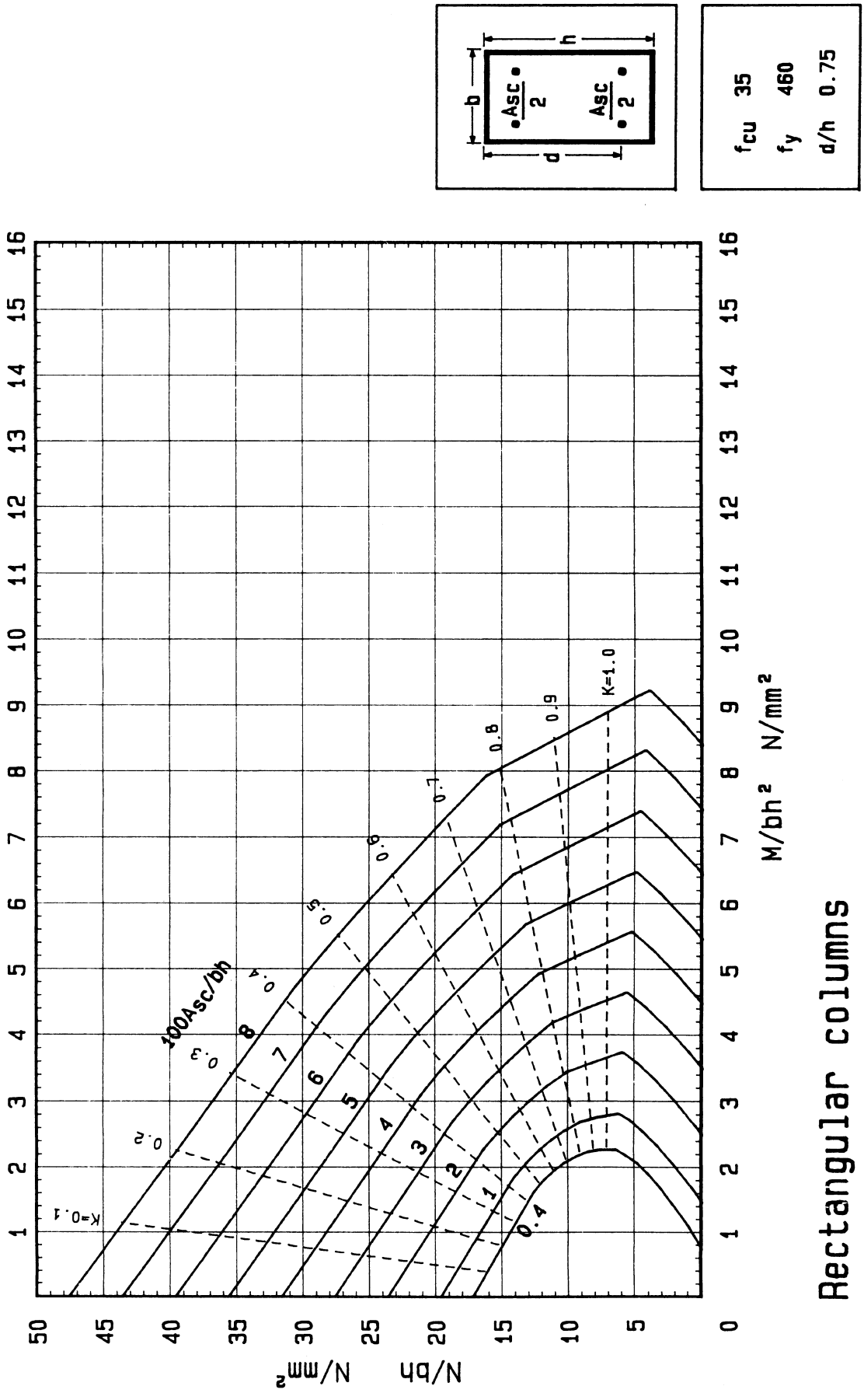


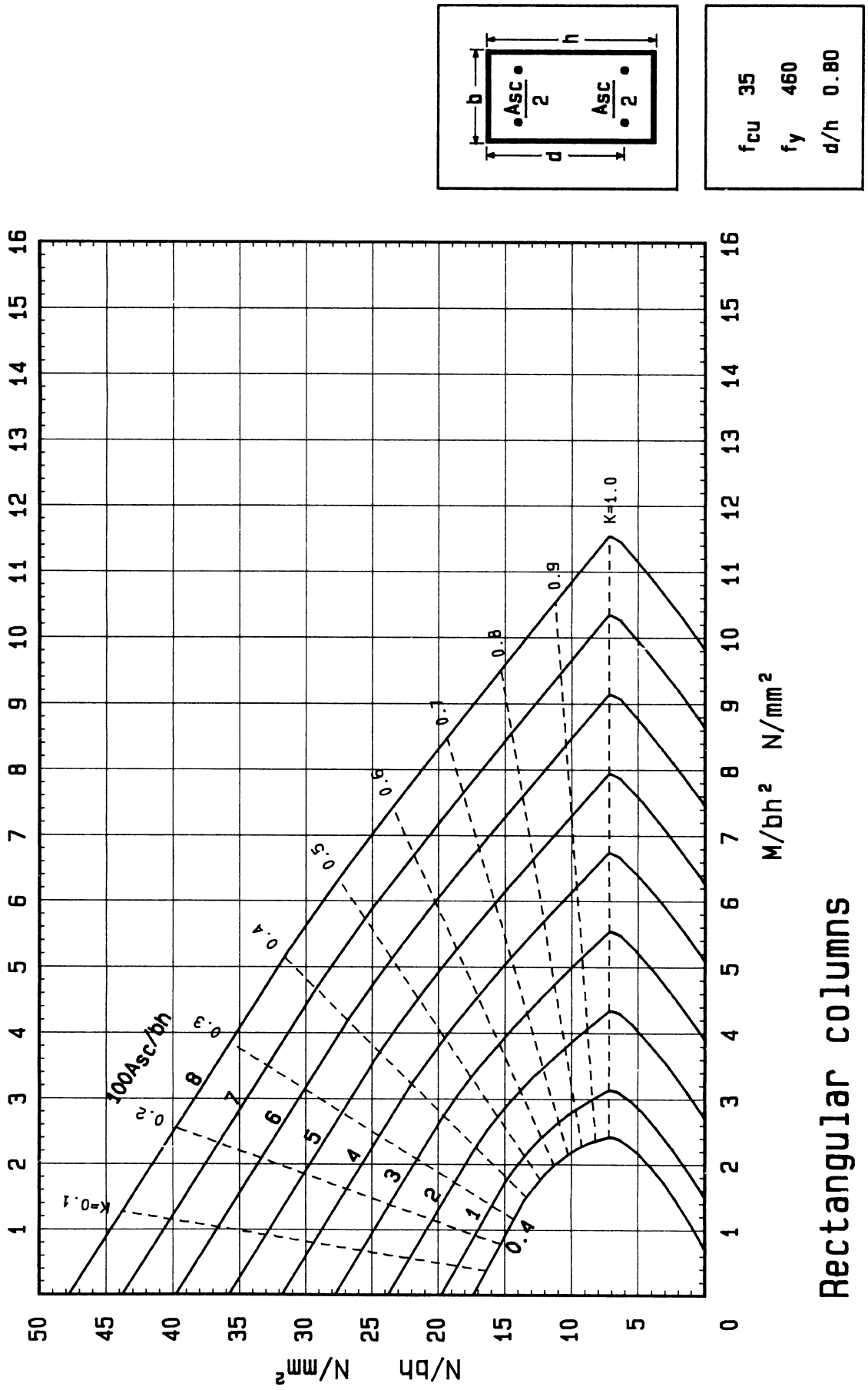


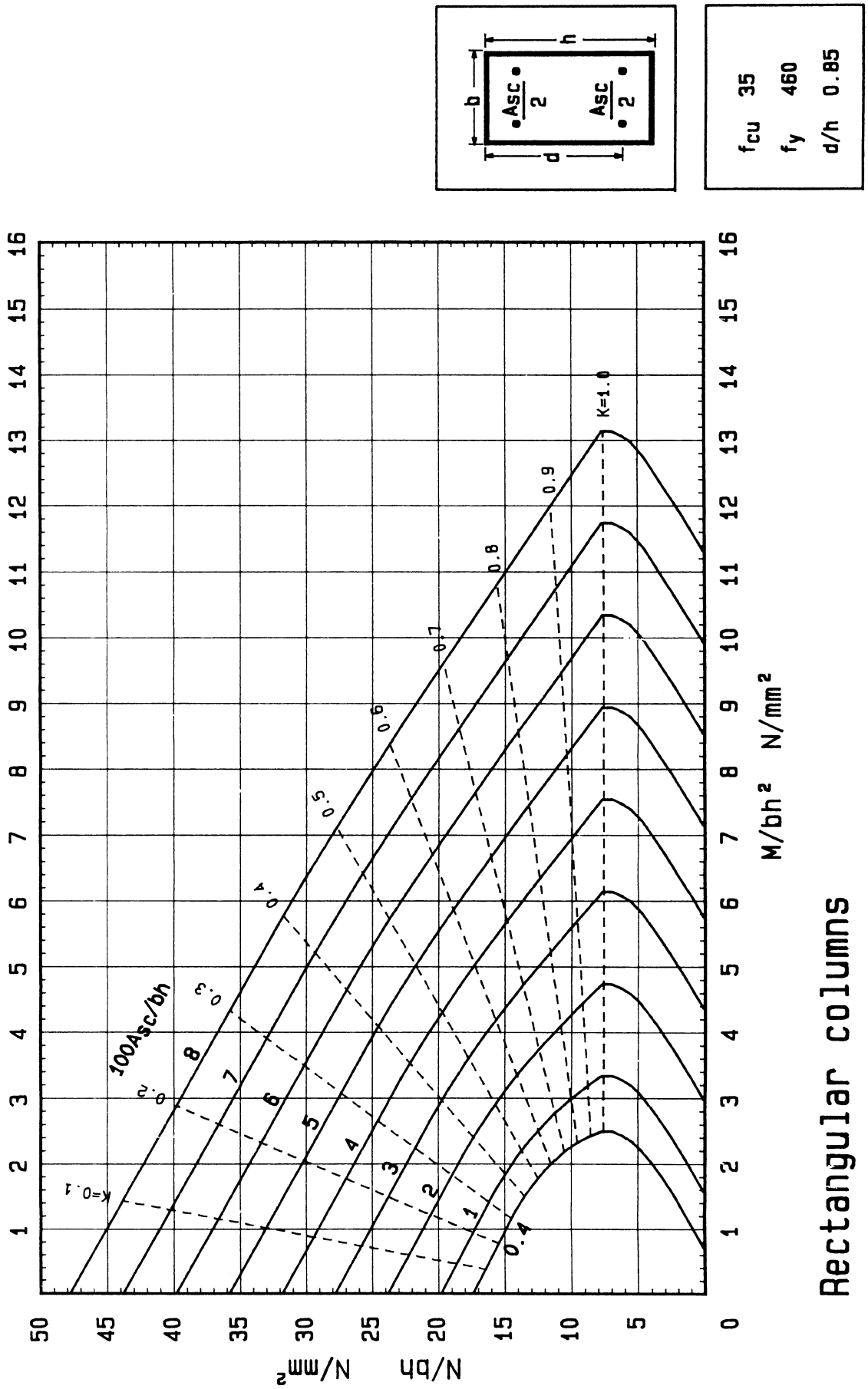
f_{cu}	30
f_y	460
d/h	0.90

Rectangular columns









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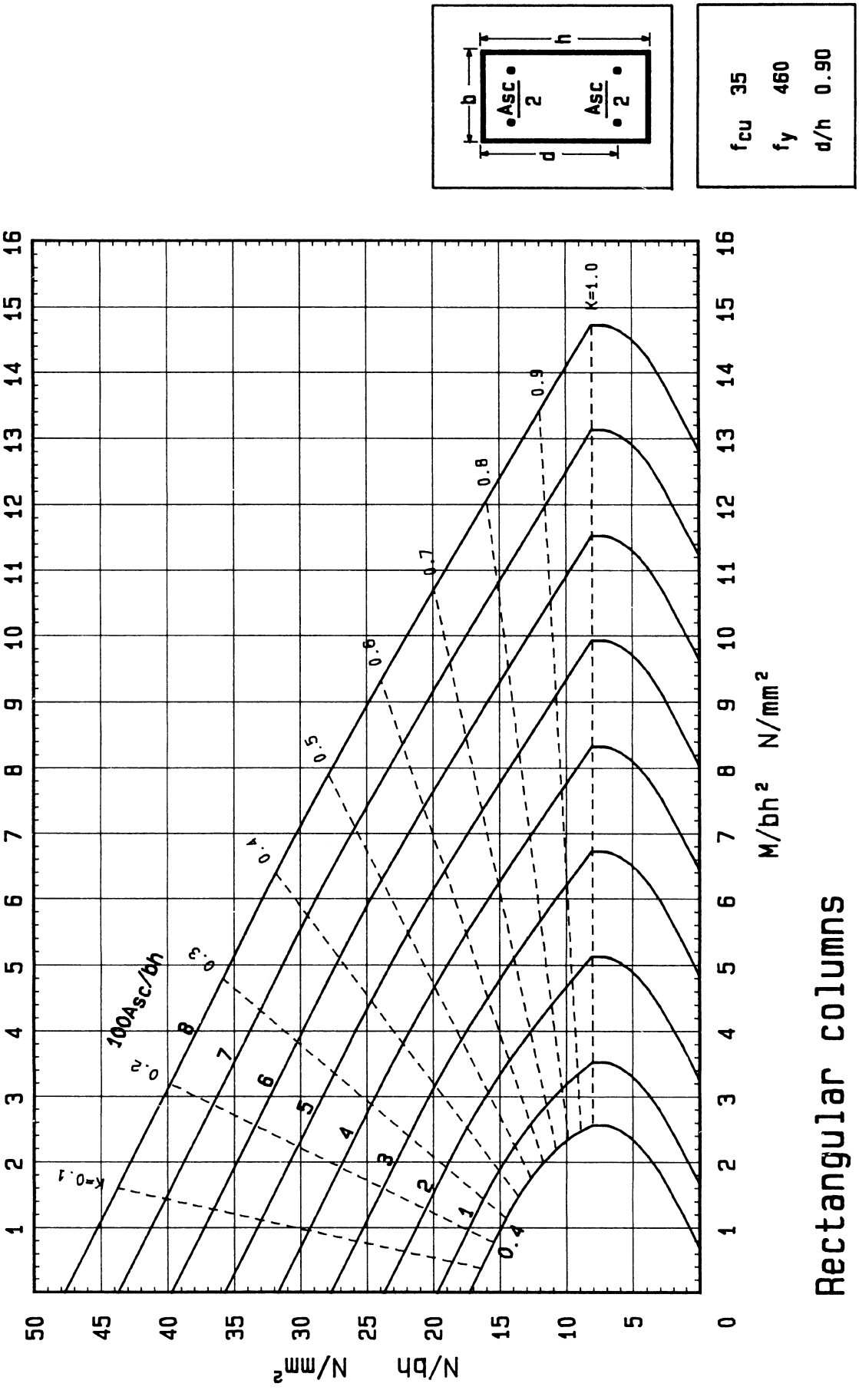

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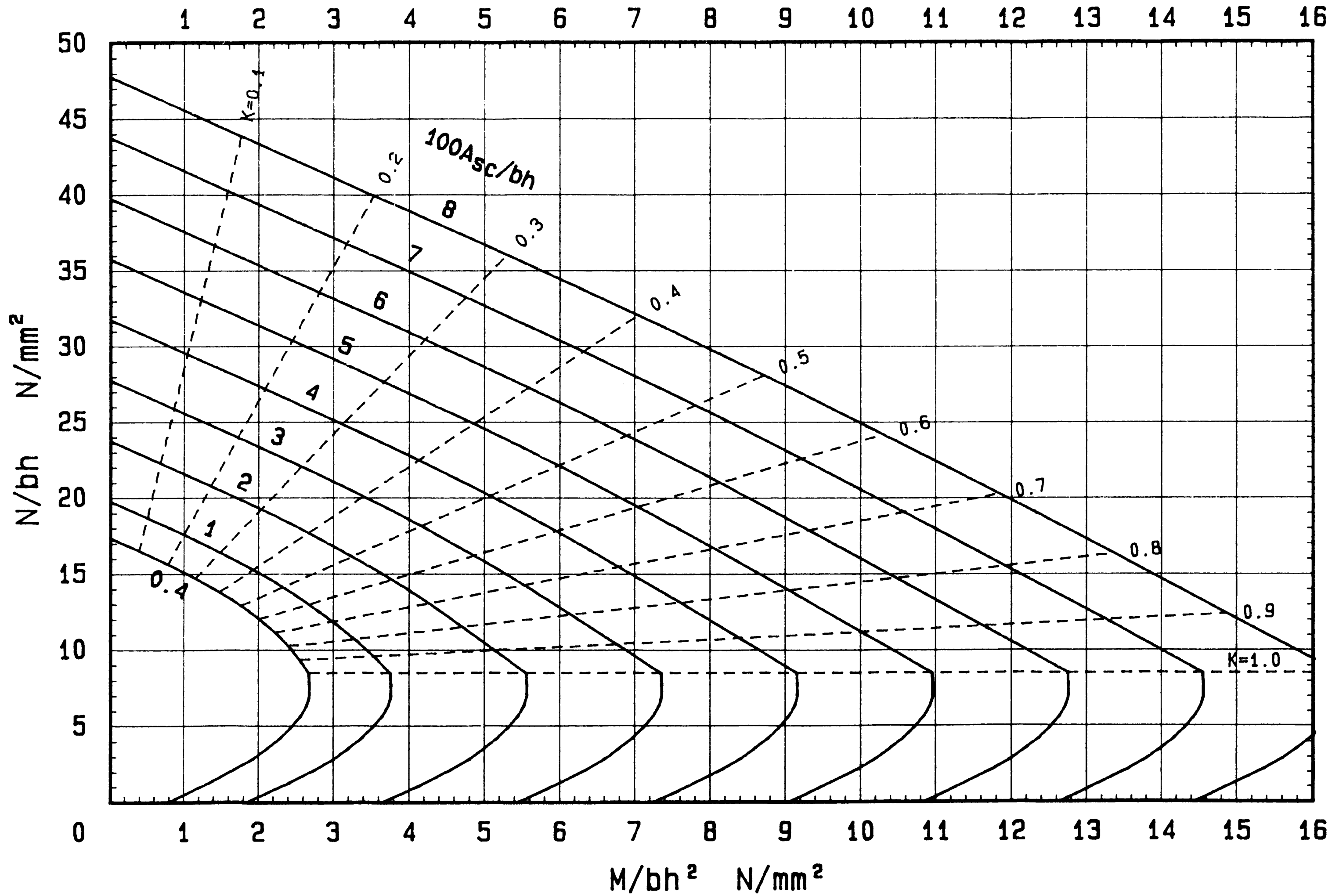
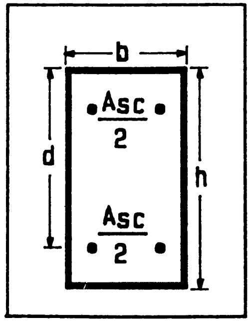
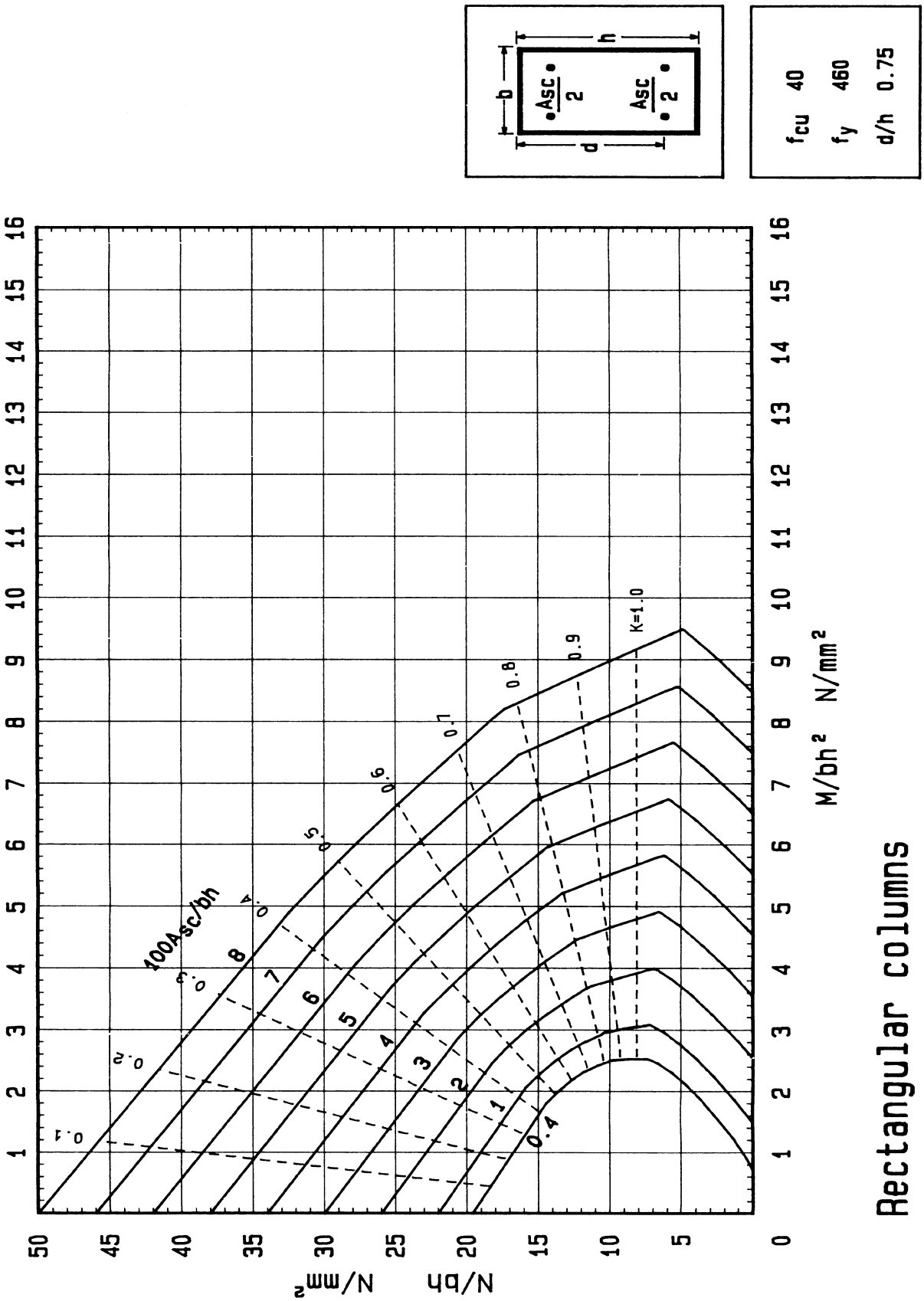
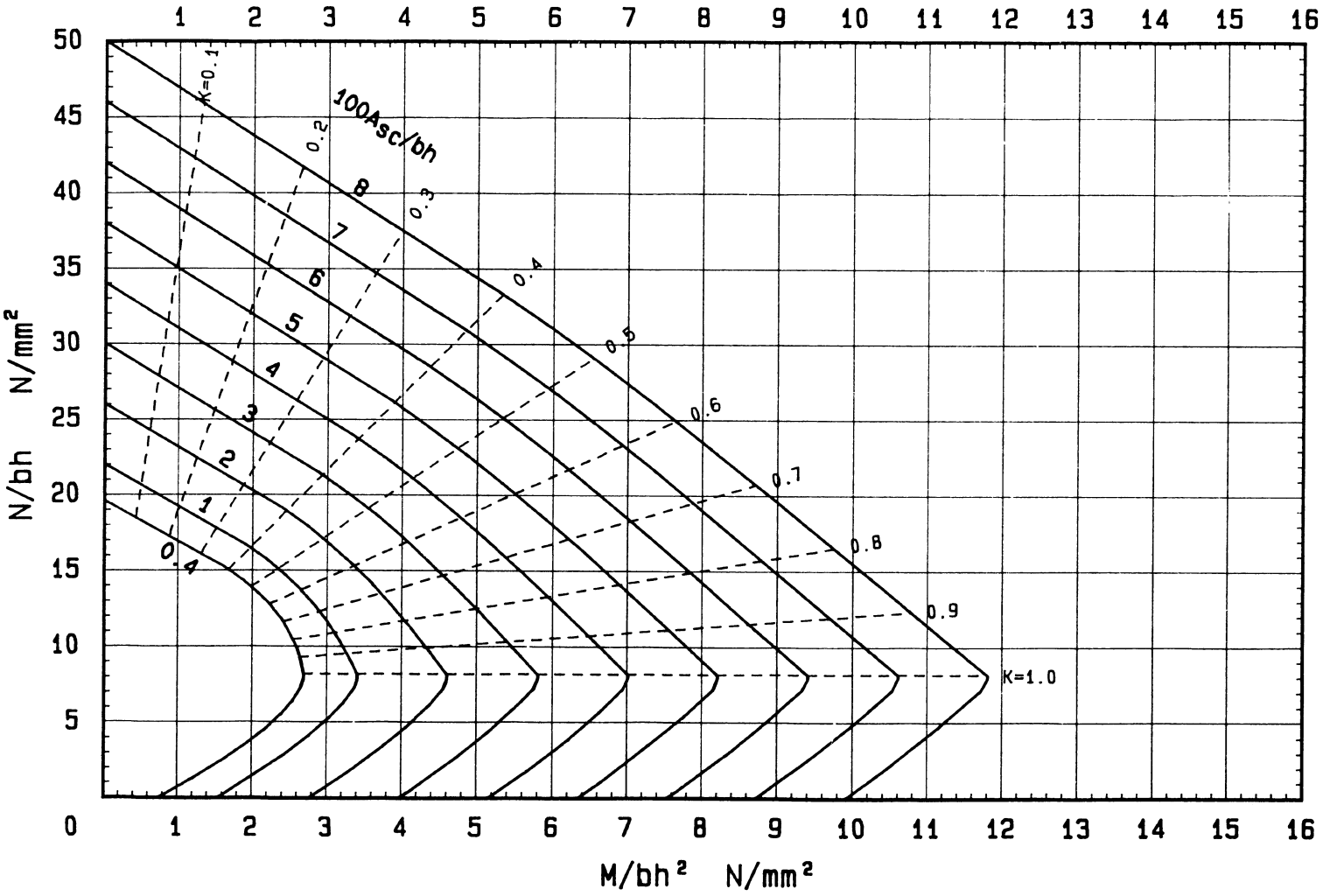


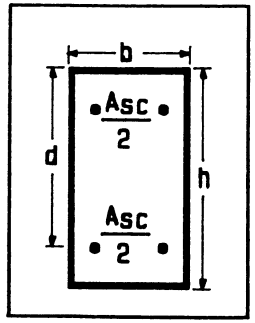


f_{cu}	35
f_y	460
d/h	0.95

Rectangular columns



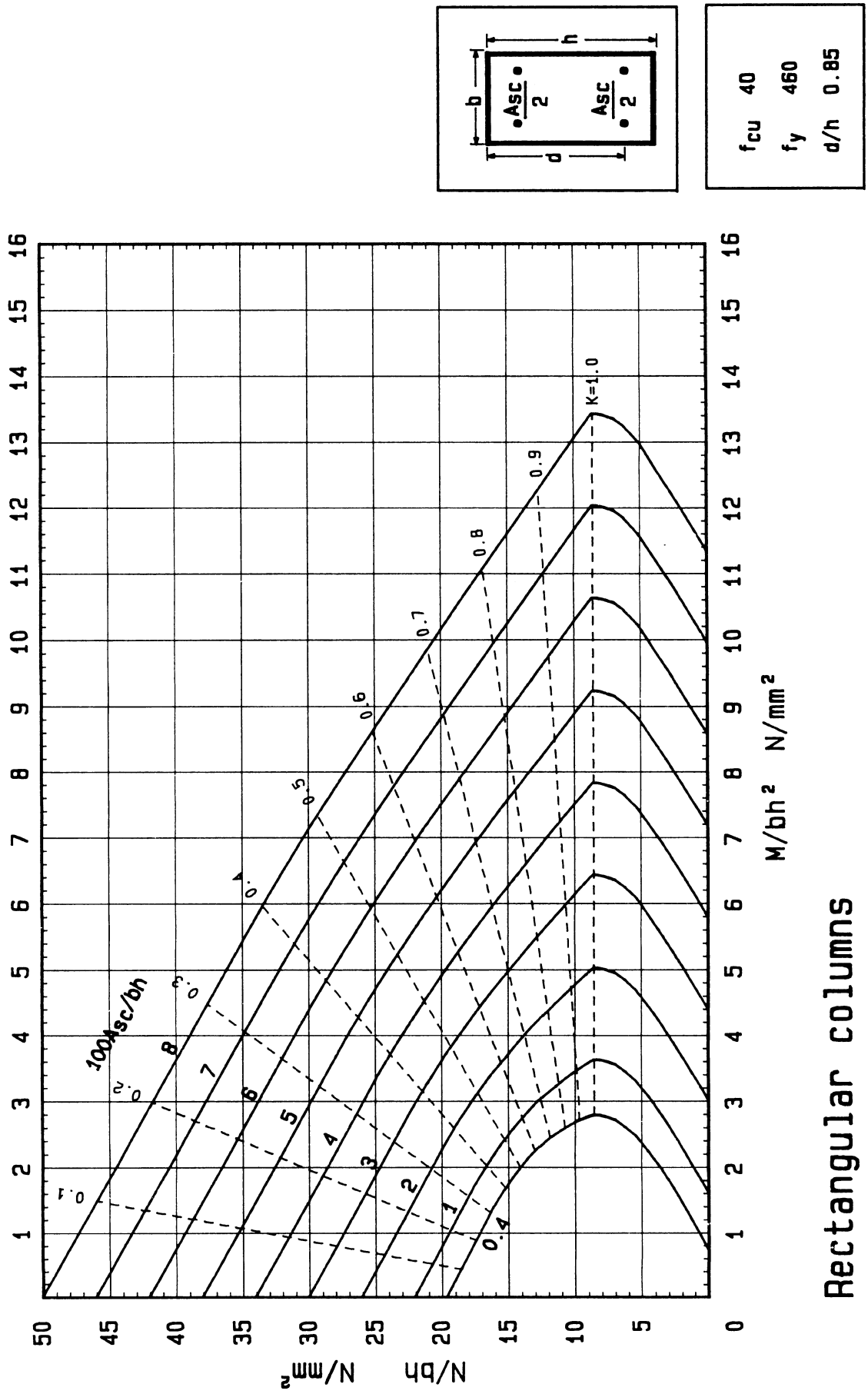




f_{cu}	40
f_y	460
d/h	0.80

Chart No. 37

Rectangular columns



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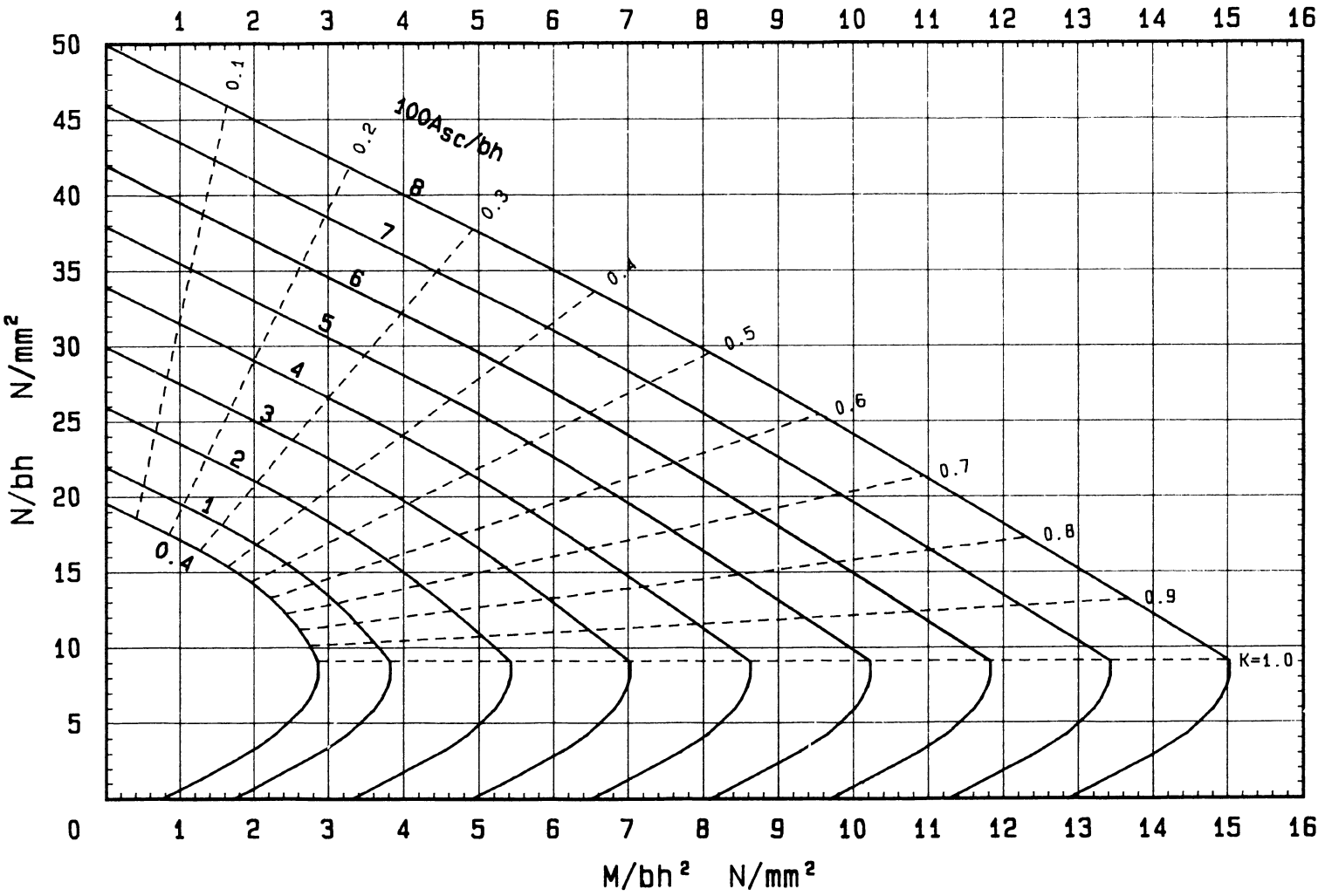

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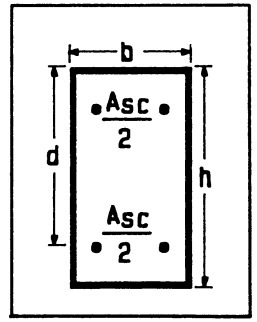
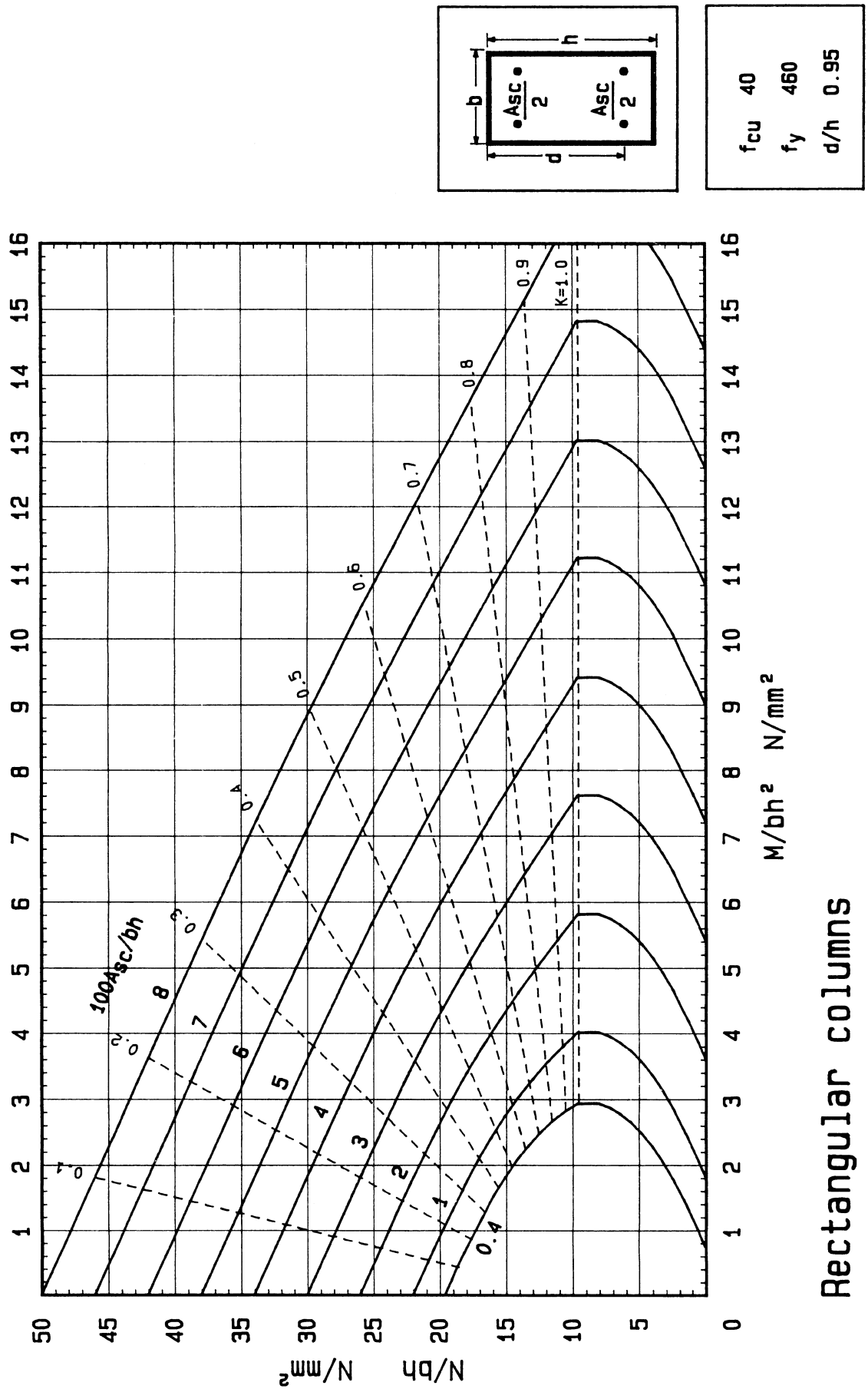
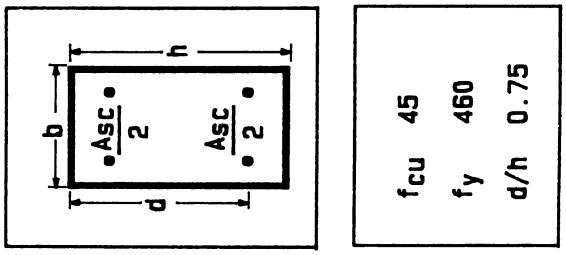
Rectangular columns



f_{cu}	40
f_y	460
d/h	0.90





f_{cu} 45
f_y 460
d/h 0.75

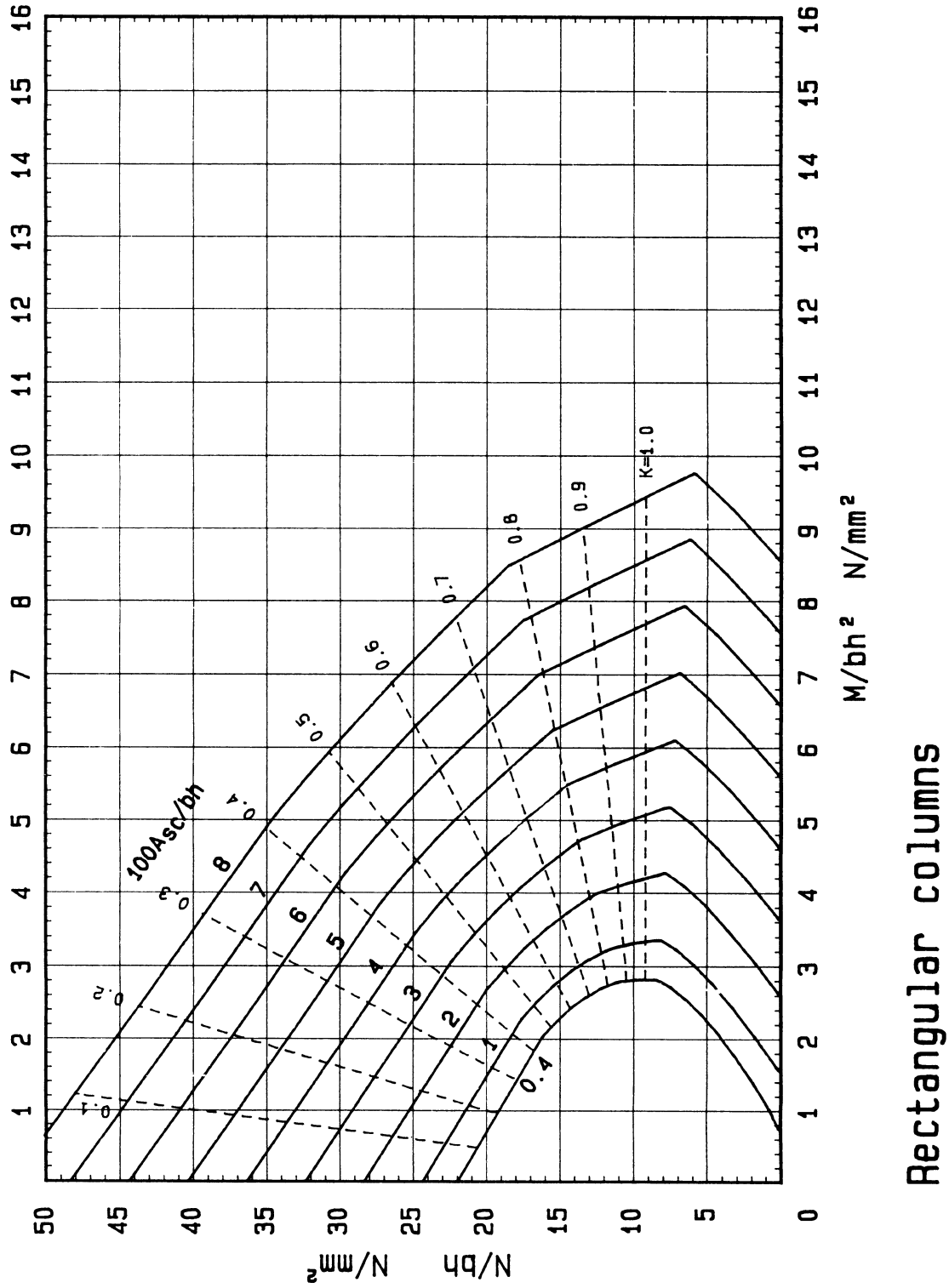
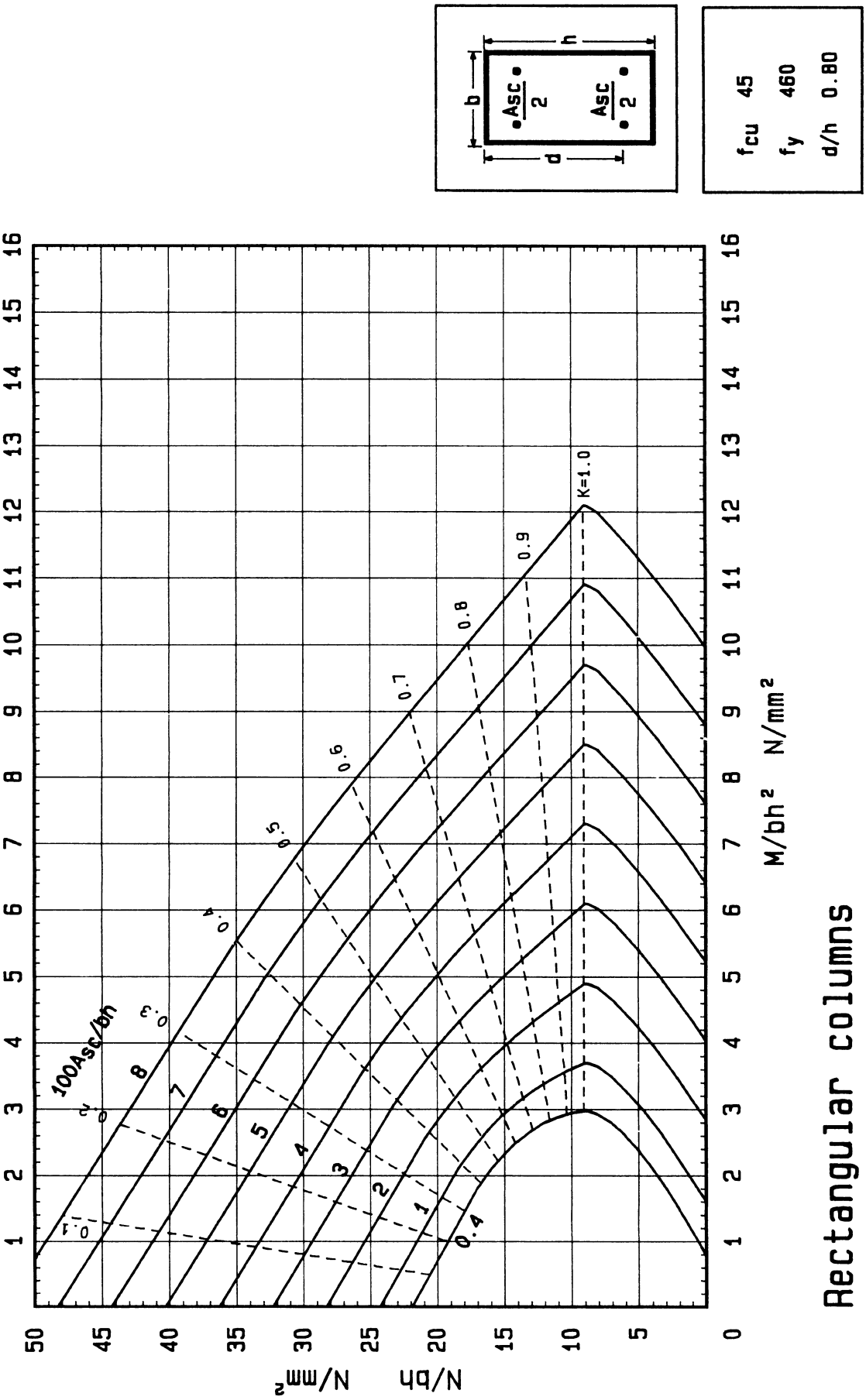
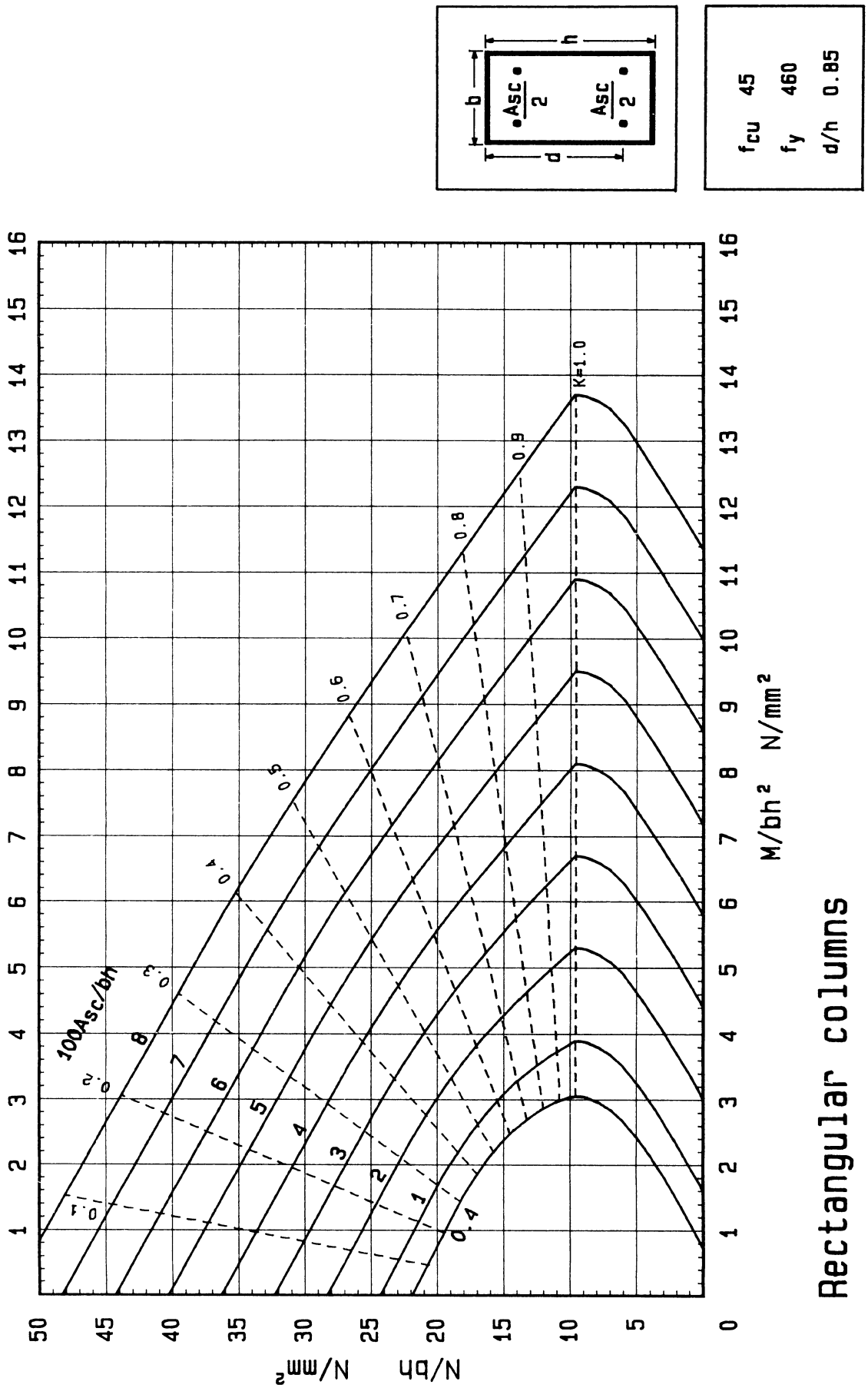


Chart No. 41





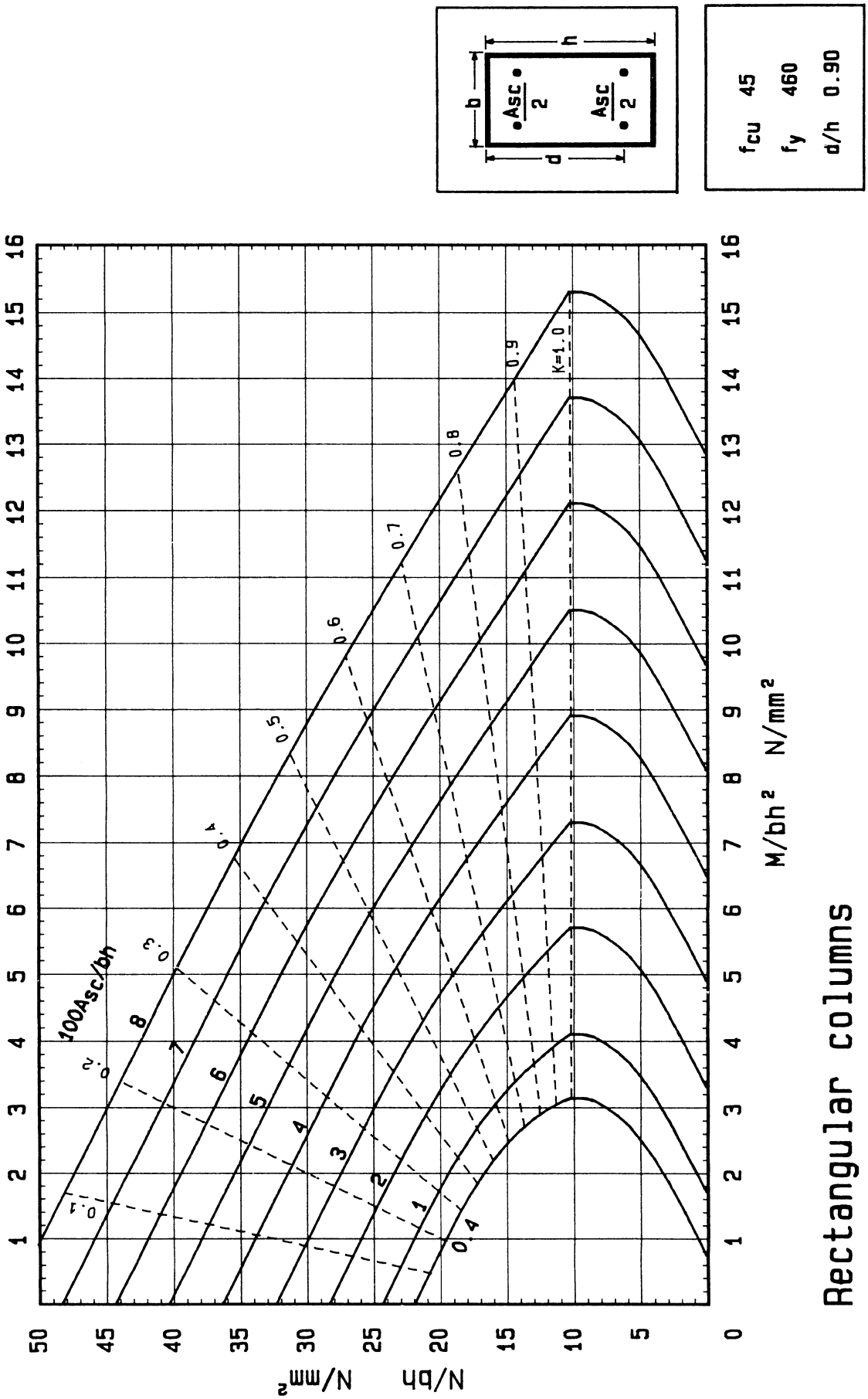
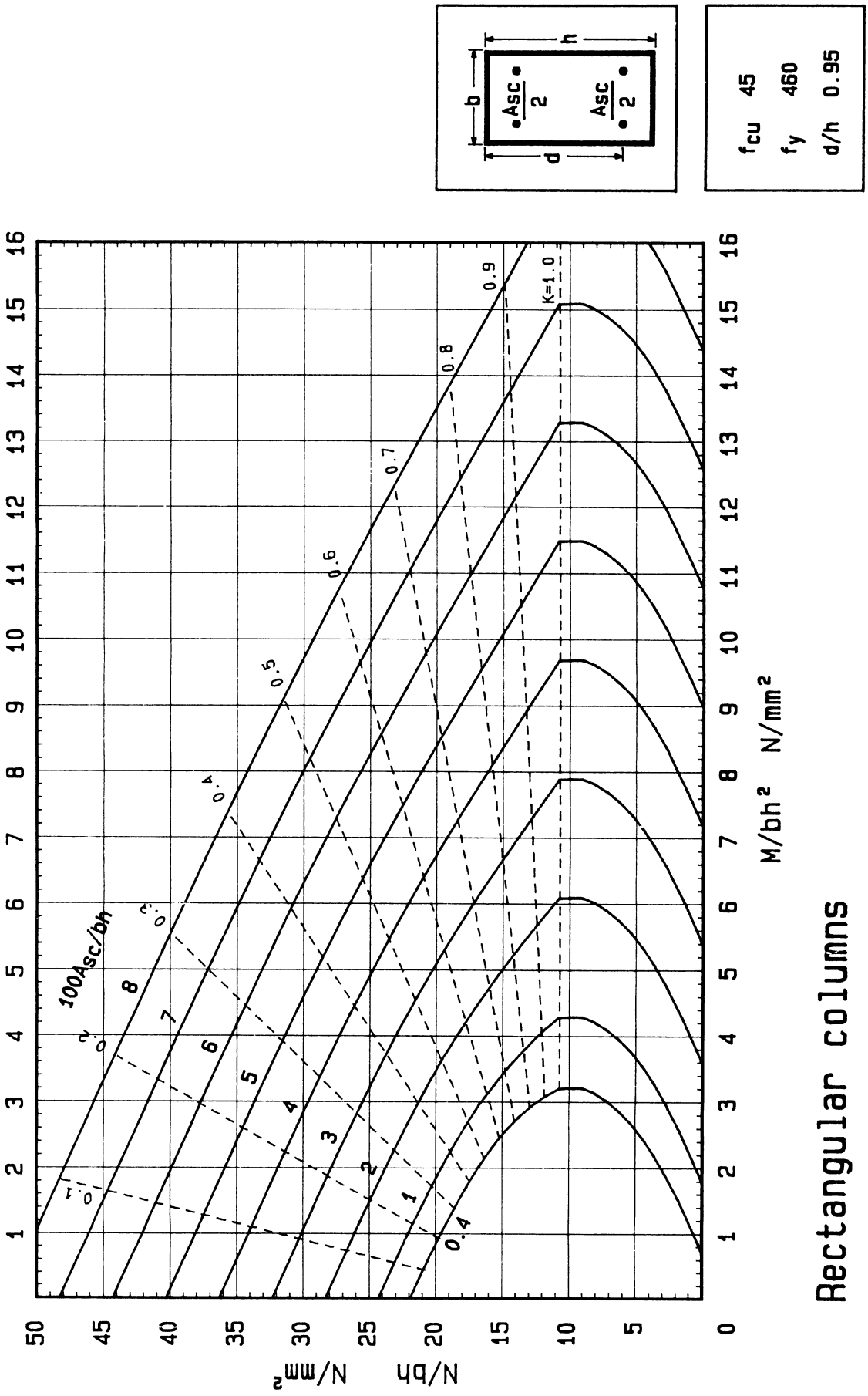


Chart No. 44



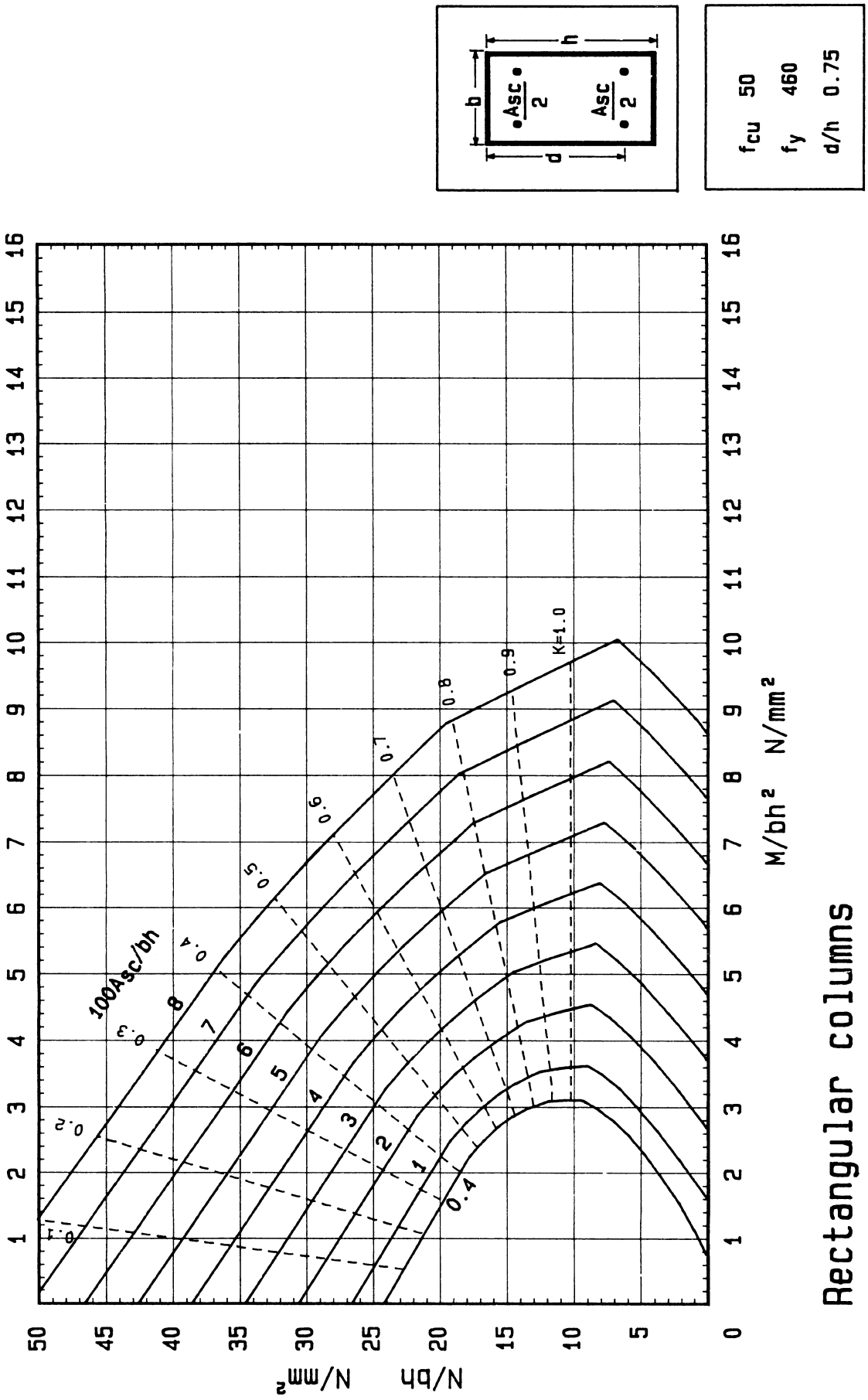
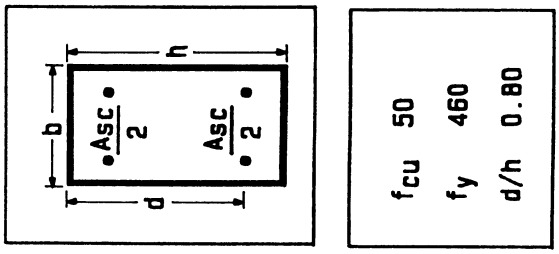


Chart No. 46



f_{cu} 50
f_y 460
d/h 0.80

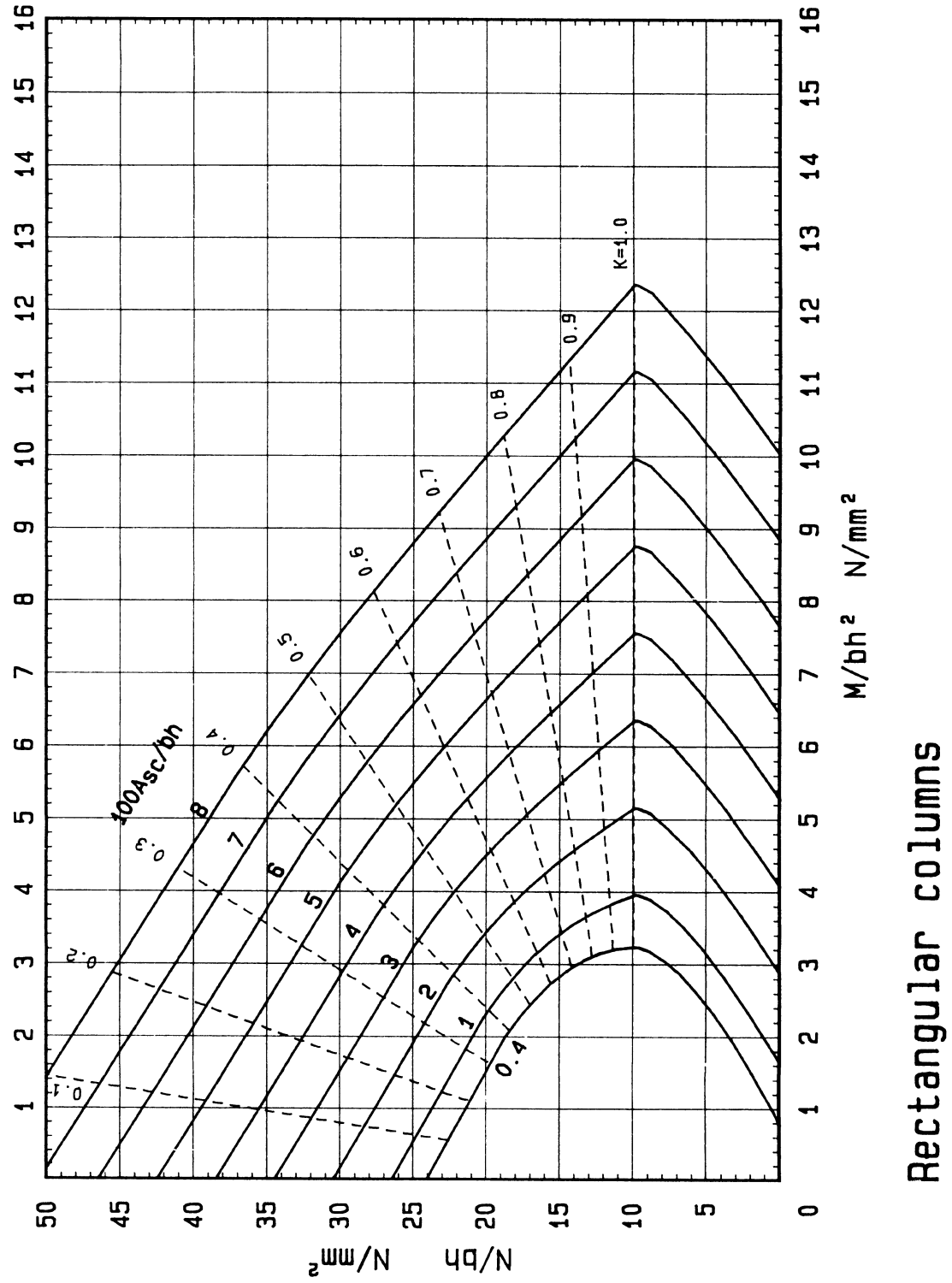


Chart No. 47

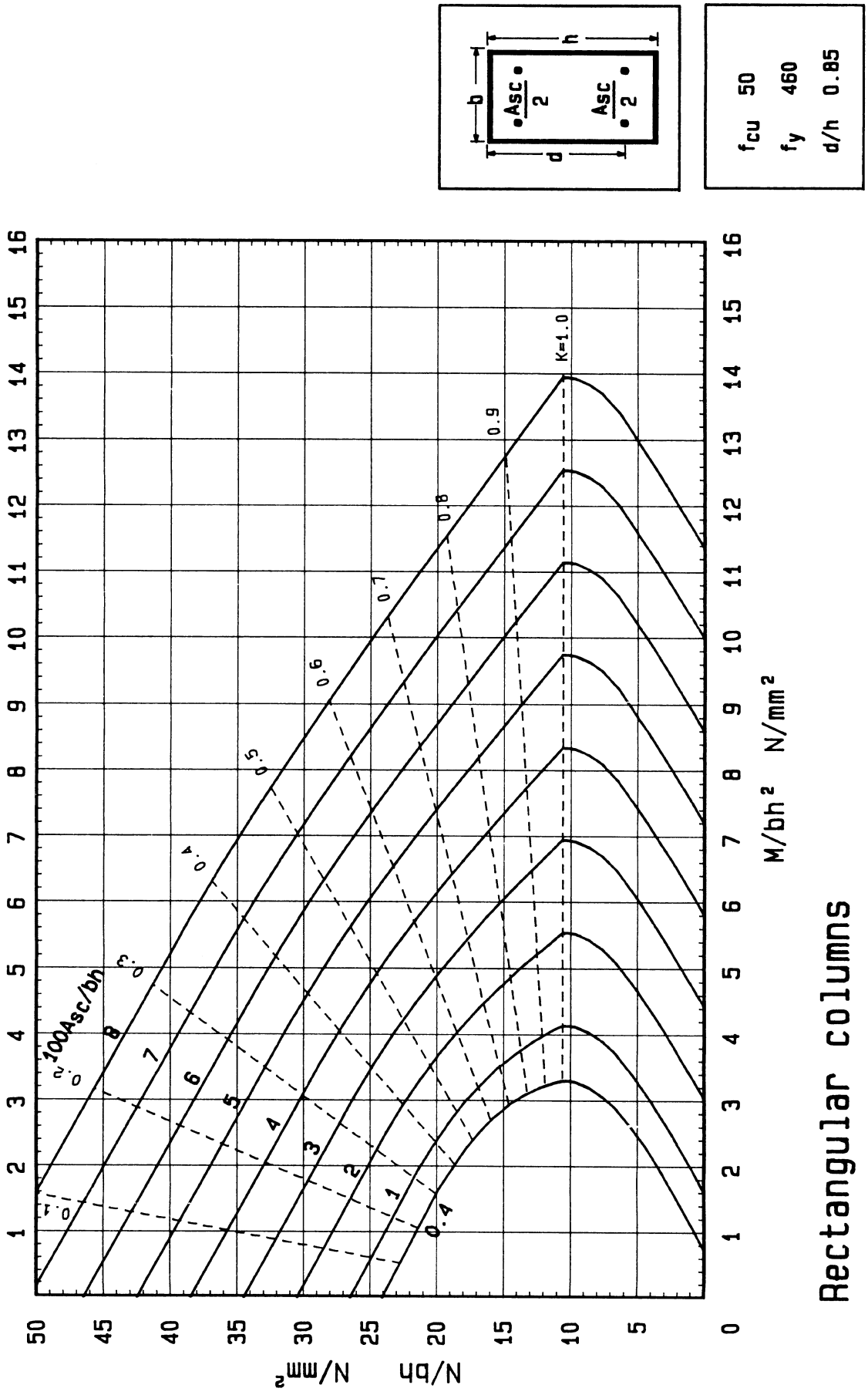
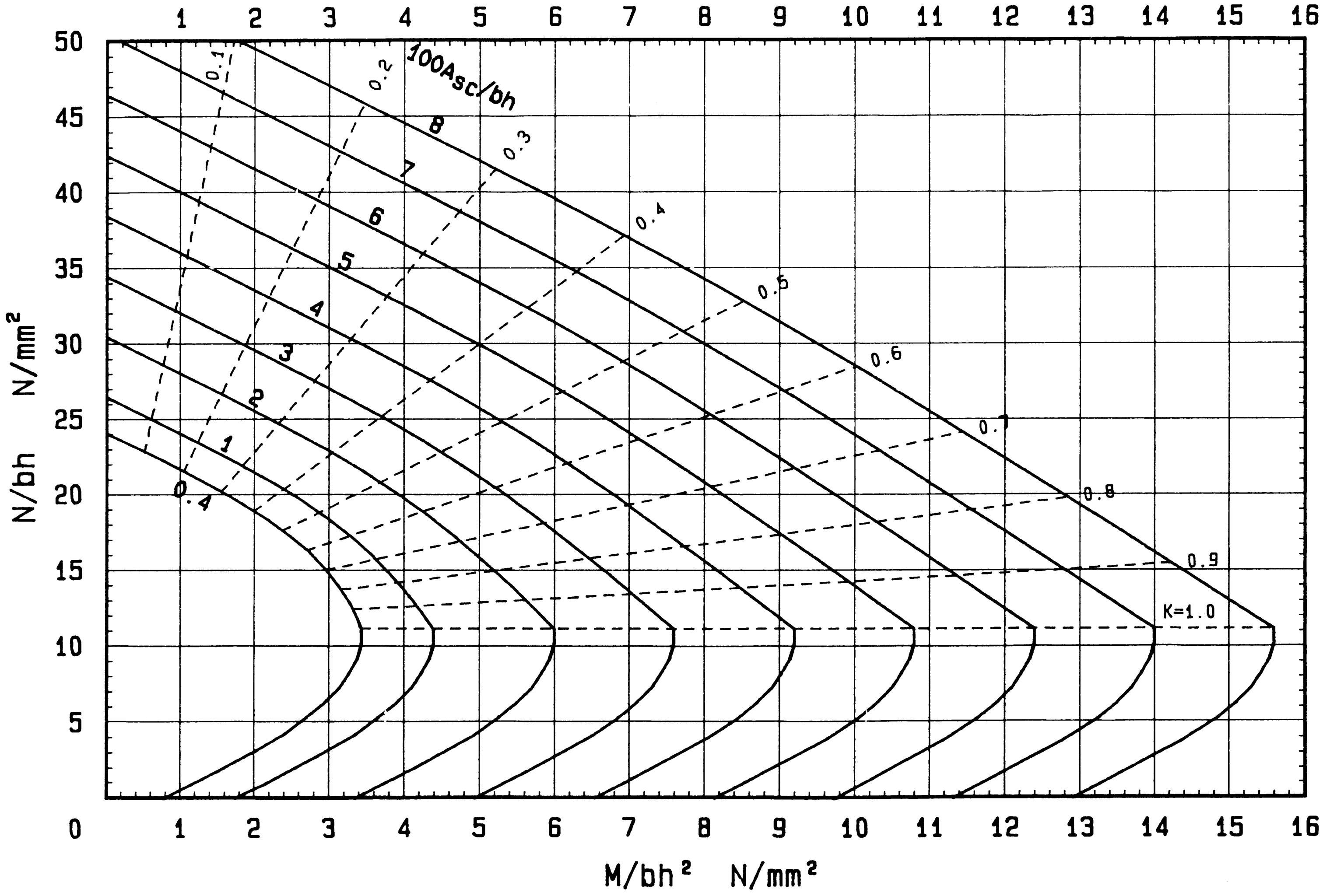
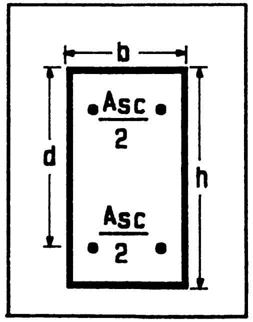


Chart No. 49



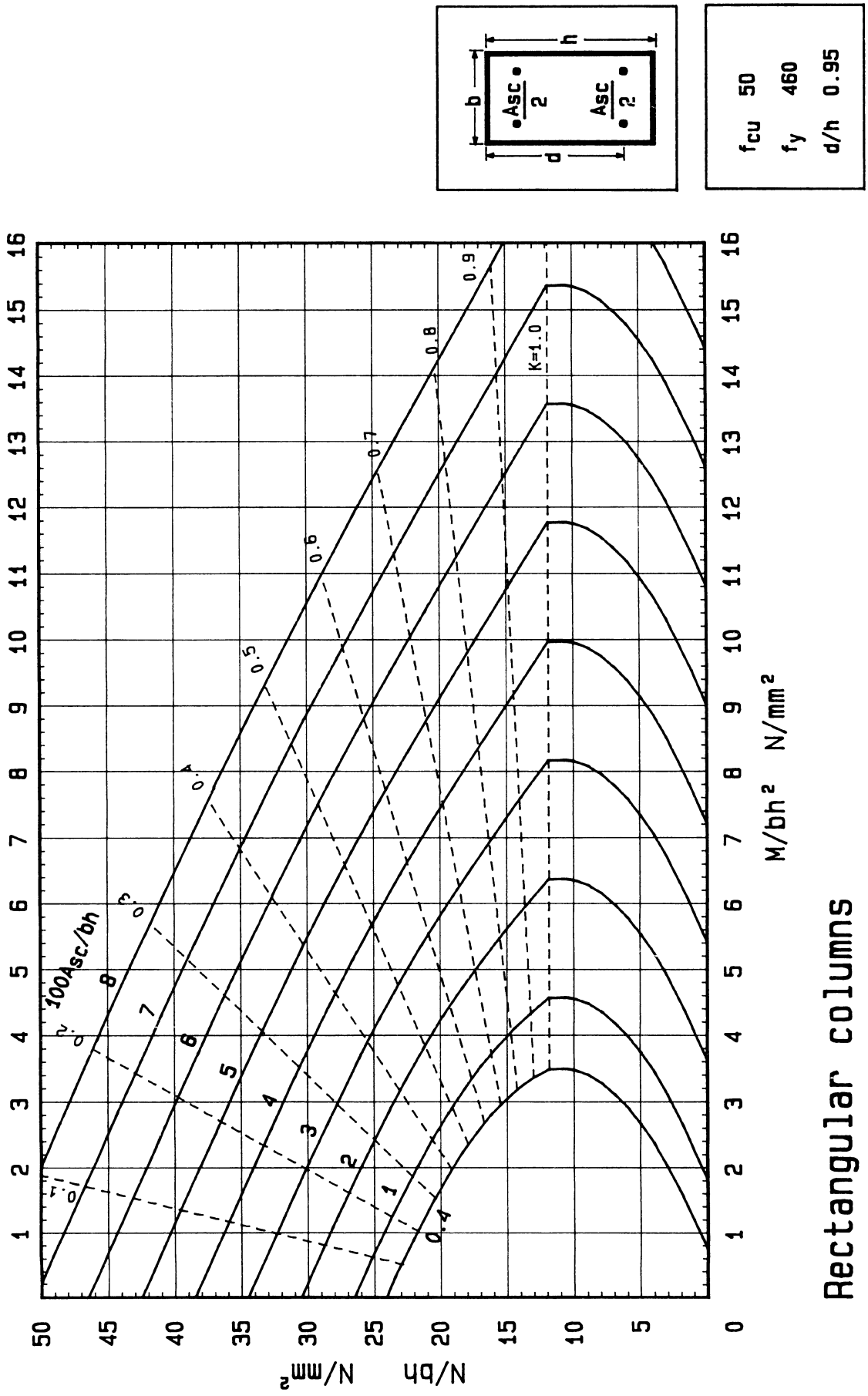
Rectangular columns



f_{cu} 50

f_y 460

d/h 0.90



Appendix A Notes on the derivation of the design charts

All the charts have been derived using the assumptions given in **3.4.4.1** of BS 8110-1:1985 for the analysis of cross sections. These assumptions are shown diagrammatically in Figure 1 to Figure 3. Figure 2 shows the stress-strain curves assumed for the concrete and Figure 3 the stress-strain relationship for the reinforcement at the ultimate limit state. In deriving the charts from these stress-strain curves, γ_m has been taken as 1.5 and 1.15 for concrete and reinforcement, respectively. For rectangular cross sections where the neutral axis remains within the section, expressions can easily be derived to give the total compressive force developed in the concrete and the position of the centroid of this force at failure. These are as follows.

Concrete compressive force at failure is given by:

$$\frac{0.45 f_{cu} bx}{0.0035}\left(0.0035 - \frac{\epsilon_0}{3}\right) = k_1 bx$$

Distance from compressive face of section to centroid of concrete compressive force is given by:

$$\left[\frac{(2 - \epsilon_0/0.0035)^2 + 2}{4\,(3 - \epsilon_0/0.0035)}\right] x = k_2 x$$

These expressions were used in the production of all the design charts for rectangular cross sections. For rectangular column cross sections where the eccentricity of the load is sufficiently small to cause the neutral axis to lie outside the section, the result obtained from the expressions is reduced to allow for the effect of the part of the parabola lying outside the section.

The figures from which the charts were drawn were produced by solution of the equations for equilibrium of moments and forces for successive values of neutral axis depth. For a doubly reinforced rectangular cross section with the neutral axis within the section, these equations are as follows.

Equilibrium of axial forces, N, is given by:

$$N = k_1\, bx + A_s'\, f_{sc} + A_s\, f_{st}$$

Equilibrium of moments, M, is given by:

$$M = k_1\, bx\,(h/2 - k_2 x) + A_s'\, f_{sc}\,(h/2 - d') + A_s\, f_{st}\,(h/2 - d)$$

For the beam charts, areas of tension steel and moments were calculated for specified areas of compression steel and neutral axis depth. For the column charts, moments and axial loads were calculated for specified steel areas and neutral axis depths.

Three assumptions of a minor nature have been introduced in the production of the charts in addition to those given in BS 8110-1.

a) No solutions have been produced for situations where $x < d'$. For the beam charts the first point calculated is for $x = d'$. This point was joined to the origin by a straight line. This is conservative.

b) Where compression reinforcement is present the full area of concrete in compression has still been assumed active, thus no reduction has been made in the calculations for the concrete displaced by the reinforcement.

c) There should, rigorously, be a slight deviation from a smooth curve of the interaction diagrams for columns in the region of low moments. This arises where the neutral axis lies just outside the section and part of the parabolic section of the stress-strain curve is curtailed. The curves have been "smoothed" in this region. The error introduced is negligible.

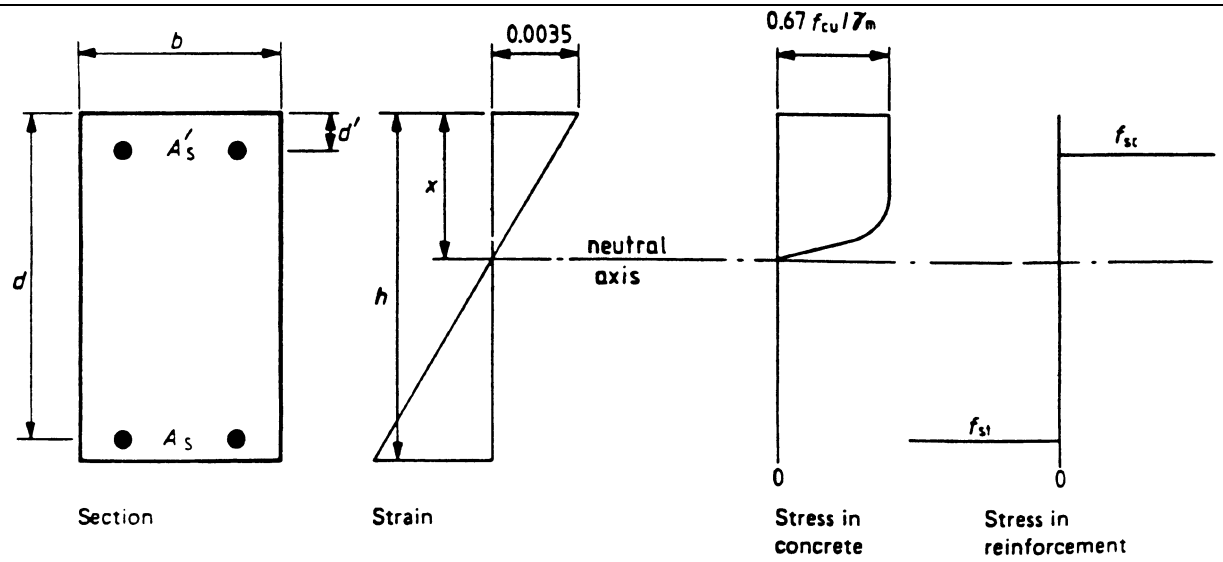
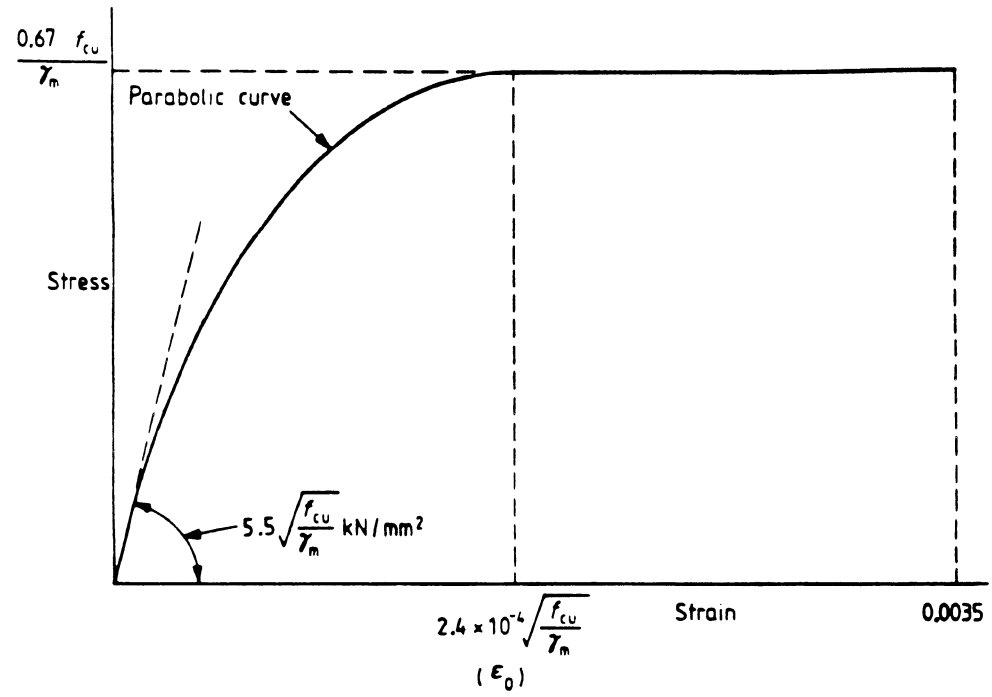


Figure 1 — Conditions of stress and strains assumed at the ultimate limit state



0.67 takes account of the relation between the cube strength and the bending strength in a flexural member. It is simply a coefficient and *not* a partial safety factor.

NOTE. f_{cu} is in N/mm².

Figure 2 — Short term design stress-strain curve for normal-weight concrete

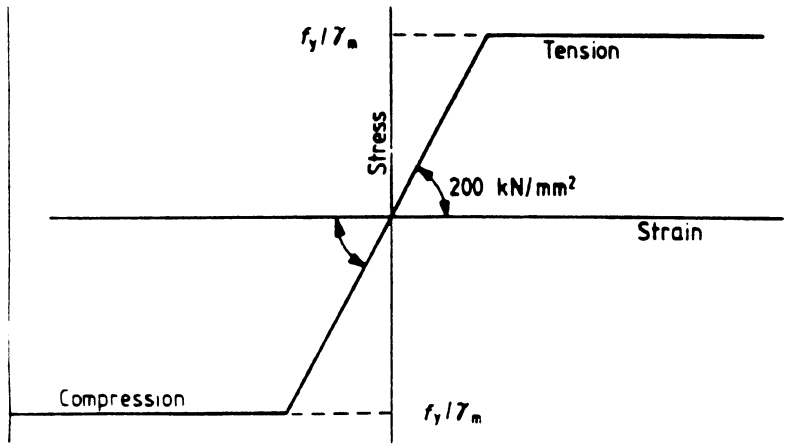


Figure 3 — Short term design stress-strain curve for reinforcement

Appendix B Design example

NOTE The use of each type of chart is illustrated by an example.

B.1 Singly reinforced beams

Design the cross section of a slab for the ultimate moment given. Design details:

f_{cu} = 25 N/mm^2;

f_y = 250 N/mm^2;

h = 160 mm;

cover = 30 mm to main reinforcement;

M per metre width = 27.6 kN m.

Assume 16 mm bars

Therefore

$$d = 160 - \left[30 + \frac{16}{2}\right]$$

$$= 122 \text{ mm}$$

$$\frac{M}{bd^2} = \frac{27.6 \times 10^6}{1000 \times 122^2}$$

$$= 1.86 \text{ N/mm}^2 \text{ per metre width}$$

From Chart No. 1

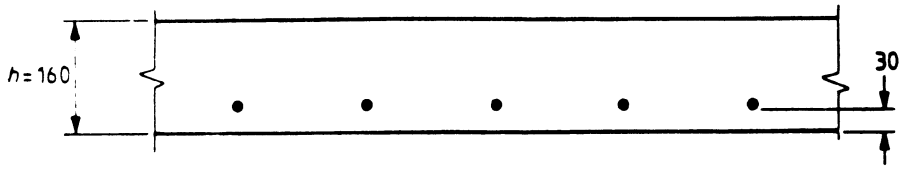
$$\frac{100 A_s}{bd} = 0.94$$

Therefore

$$A_s = \frac{0.94 \times 1000 \times 122}{100}$$

$$= 1145 \text{ mm}^2 \text{ per metre width}$$

Therefore a suitable solution is to use 16 mm bars at 175 mm centres (i.e. 1 150 mm^2 per metre width).



B.2 Doubly reinforced beams

Design the centre cross section of a rectangular beam to comply with the following requirements at the ultimate limit state.

Design details:

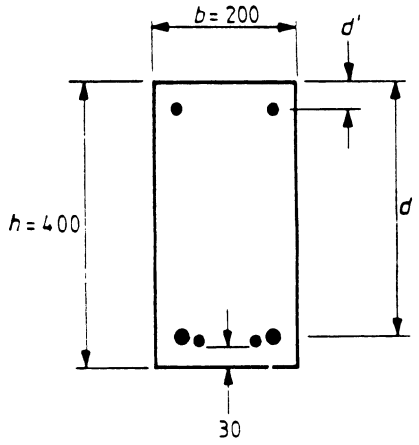
f_{cu} = 40 N/mm^2;

f_y = 460 N/mm^2;

M = 127 kN m;

cover = 30 mm to main reinforcement.

The critical cross section under consideration is given below and the bending moment at the ultimate limit state has been obtained by redistribution involving a reduction equal to 30 % of the numerically largest moment given by the elastic maximum moments diagram for the beam.



Assume 10 mm bars for compression reinforcement and 25 mm bars for tension reinforcement.

$\frac{d'}{d} = \frac{35}{357.5} \approx 0.10$ therefore use Chart No. 18

M/bd^2 = 4.97

A reduction of 30 % due to redistribution requires that the neutral axis depth should be limited such that $x/d \leqslant 0.3$. Thus a solution must be sought on or to the left of the line marked x/d = 0.3 on Chart No. 18. A suitable solution is:

100 A_s/bd = 1.42;

100 A_s'/bd = 0.3 (by interpolation).

Therefore A_s = 1 015 mm^2

and A_s' = 214 mm^2

Therefore reinforcement to be provided

A_s: 2 No. 20 + 2 No. 16 (area 1 030 mm^2)

A'_s: 2 No. 12 (area 226 mm^2)

The use of 12 mm compression reinforcement means that d'/d is now 0.101 thus the use of Chart No. 18 is still justified.

B.3 Rectangular columns

Design the cross section of a rectangular column to comply with the following requirements at the ultimate limit state.

Design details:

f_{cu} = 50 N/mm^2;

f_y = 460 N/mm^2;

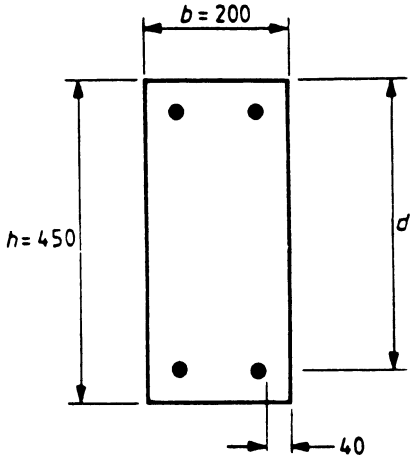
cover = 40 mm to main reinforcement;

M = 91.2 kN m about major axis

0 kN m about minor axis

N = 2 460 kN

The cross section of the column is as follows:



Assume 32 mm bars

$\frac{d}{h} = \frac{394}{450} \approx 0.88$ therefore use Chart No. 49 where

d/h = 0.90

M/bh^2 = 2.25

N/bh = 27.4

From Chart No. 49

100 A_{sc}/bh = 2.6

Therefore A_{sc} = 2 340 mm^2

Therefore a suitable solution is to use 4 No. 32 mm cold worked bars (i.e. 3 220 mm^2).

Publications referred to

BS 8110, *Structural use of concrete.*
BS 8110-1, *Code of practice for design and construction.*
BS 8110-2, *Code of practice for special circumstances.*

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